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Calculation of three-layer plates by methods of vibration theory

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Abstract: A three-layer plate with thick hard outer layers and a thin soft inner layer was studied. A model is considered on the example of an anti-sandwich panel to describe the mechanical behavior of a plate on the example of a solar panel. A review of the scientific literature was conducted, in which models of both analytical and numerical methods for calculating three-layer plates are displayed. The scientific work uses the method of finite element analysis using a spatial shell element, as well as the theory of single- and multi-layer plates. These elements combine the topology of volumetric elements and the kinematic and structural equations of a classical shell element. Shell elements based on continuum mechanics were used for numerical simulation. The study was carried out under static load under different conditions, and also the self-oscillations of the anti-sandwich were analyzed using the theories of Kirchhoff and Reisner-Mindlin. As part of the scientific work, a study of the mechanical model of a thin solar panel was carried out using finite element analysis taking into account different temperature conditions and comparing the results with existing studies.
Keywords: solar panel, anti-sandwich, Kirchhoff theory, Reisner-Mindlin theory, natural frequencies, finite element method, ABAQUS.

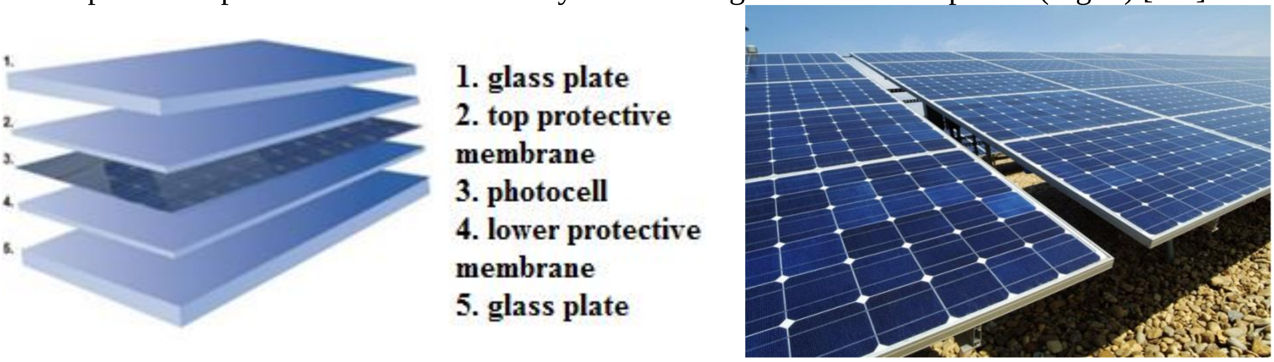
1. Introduction

Nowadays, there is a rapid development of renewable energy sources. Among them, one of the most widespread is solar energy. Solar panels are usually subjected to significant loads due to changing weather conditions. These can be loads due to wind or precipitation, as well as cyclic daily or seasonal temperature changes. This can lead to a violation of the tightness between the layers, the formation of cracks in fragile materials and, as a result, to a decrease in efficiency. Therefore, modeling and research of the behavior of individual layers, as well as of the entire plate, is a necessary and very relevant issue today [1,2]. In scientific works, solar panels are considered as multilayer plates, the study of which is carried out by classical theories.

2. Object and subject of research

The object of the study is an anti-sandwich panel - a three-layer plate with thick, hard outer layers and a thin, soft inner layer.

A typical structure of solar cells consists of outer glass layers and inner soft polymer layers, which perform a protective function for very thin and fragile silicon solar panels (Fig. 1) [3-5].



a **b**

Fig. 1 Structure of a solar panel [2]

This mechanical model describes the typical structure of solar cells, consisting of outer layers of glass and inner soft polymer layers that perform a protective function for very thin and fragile silicon solar panels.

There is very little information in the freely available literature regarding the type of solar panel fixing. In this regard, the calculation will be carried out in two limiting cases of fastening: rigid anchoring and free anchoring, which allows relative displacement of the layers.

Classic shell elements do not take into account transverse shear, which makes it impossible to model the real behavior of the solar panel in the thickness direction.

3. Target of research

The main aim is to study the model of the solar panel using the spatial elements of the shell and compare it with the existing results in order to prove the feasibility of using these elements to solve similar problems. Thus, there is a need to formulate the main theoretical statements regarding the continuum mechanics, on the basis of which two researched approaches are built: the theory of multilayer plates and the spatial elements of the shell. It is also relevant to determine the natural frequencies of vibration of the plate by classical methods of the theory of oscillations using the finite element method.

One of the research issues is the analysis of the basic theory of the spatial shell element, which includes the following points:

• study of convergence of discretization in order to analyze the accuracy of the results

• calculation of deflection of the structure in the direction of thickness and movements in planar directions

• study of the structure during the change of the mechanical characteristics of the middle layer (change of the modulus of elasticity)

• comparative analysis of results with LWT results

• asymptotic analysis in two limiting cases using the Kirchhoff and Reisner-Mindlin theory in order to check the correctness of the results.

• research on the dependence of the natural frequencies of the structure on the change in the mechanical characteristics of the middle layer

4. Literature analysis

From a mechanical point of view, such a solar panel structure can be considered as a multilayer composite with isotropic properties of each layer. Since the solar elements are very thin, and almost do not affect the overall stiffness of the entire plate, they can be neglected. Thus, a three-layer thin composite with rigid outer layers (glass) and a very thin, soft and shear-yielding inner layer is considered. Due to specific geometric and mechanical characteristics, such a mechanical structure was called an anti-sandwich [6-8, 14-16]. Thus, the anti-sandwich is a mechanical model that reflects the real geometry and is able to describe the mechanical behavior of the solar panel.

To date, studies of multilayer structures for use in solar cells are presented. For example, in works [3, 9, 14-17], three-layer glass beams were investigated based on the theory of laminates and the theory of first-order shear deformations (FSDT).

In work [6], a finite-element analysis of an anti-sandwich panel was performed based on the laminate theory (layerwise theory, LWT) with the use of a specially developed finite element. As a result, displacement graphs and the dependence of maximum deflections on the modulus of elasticity of the middle layer were obtained.

In the papers [7, 13] at the Department of Mechanics of the Faculty of Mechanical Engineering at Otto-von- Gerike University of Magdeburg a new finite element (UEL) based on multilayer theory was programmed by [9]. With the help of this element, three-layer plates were calculated under different fastening conditions. In order to verify the accuracy of the calculation, the narrow plate was investigated using a newly developed element and a classical volume element. In addition, all numerical calculations were compared with the analytical LWT solution derived for a three-layer plate with a soft inner layer.

Thus, using these elements, it is possible to automate the process of creating a finite-element mesh for arbitrarily complex geometry. This will allow automated parametric calculations of arbitrarily complex geometric shapes [11].

In [9], asymmetric laminates with a soft inner layer were studied for modeling solar modules. The paper shows that there are significant deviations between the results of the LWT and FSDT theories for a three-layer beam with asymmetric layers. This indicates that the first-order shear theory allows large errors in the calculation results of such structures.

It has been found that the LWT theory is suitable for the calculation of multilayer structures with a very thin and soft inner layer. In works [2, 6-8, 10, 13, 20, 21], its application was extended to the calculation of multilayer plates that model the structure of solar panels.

5. Research methods

5.1 Geometry modeling

1) Research was conducted for a three-layer thin anti-sandwich panel, in which the outer layers are made of glass, the inner layer is made of soft polymer. The geometric features of this model are as follows:

2) Planar dimensions are much larger than the total thickness: $L_{1,2} > L_3$ (Fig. 2)

3) The thickness of the middle layer is very small compared to the thickness of the outer layers: $h^c < h^в, h^н$ (Fig. 2, table. 1).

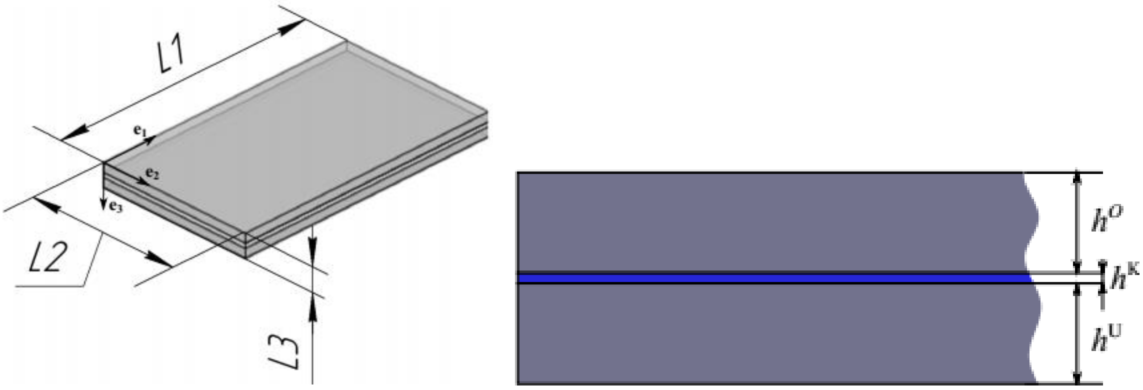


Fig. 2 Plate structure

Table 1. Geometric data [6]

Parameter	Value
$L_1[mm]$	1620
$L_2[mm]$	810
$h^{в}[mm]$	3,2
$h^{н}[mm]$	3,2
$h^{c}[mm]$	1,0

5.2 Materials selection

As already mentioned earlier, solar modules are protected by glass from the outside. Inside is a protective polymer layer, which is usually made of ethylene vinyl acetate (Ethylen-Vinylacetat-Copolymer, EVA). It compensates for various thermal and mechanical deformations and is a sealing layer for photocells [6].

An important assumption is that each layer is assumed to be isotropic. In addition, the values of the mechanical characteristics of the outer layers significantly outweigh the corresponding values of the inner layer: $E_{н} = E_{в} > E_{c}$, $G_{н} = G_{в} > G_{c}$. Materials data provided in table. 2.

Table 2. Material data [6]

Parameter	Glass	EVA
$E\left[N/mm^2\right]$	73000	7,9
$\nu\ [-]$	0,3	0,41
$\rho\left[kg/mm^3\right]$	$25\cdot10^{-7}$	$960\cdot10^{-9}$

Since the goal is to determine the mechanical behavior of the anti-sandwich under real weather conditions, research at different temperatures is important. The research was carried out at the following temperatures: under normal conditions (+23 °C), in conditions of elevated (+80 °C) and low (-40 °C) temperatures. At the same time, it is assumed that the properties of the outer layers do not change depending on the temperature, but only the mechanical characteristics of the material of the middle layer change. The corresponding values are given in table. 3.

Table 3. Data of the material of the middle layer at different temperatures [6]

$T\,[°C]$	**-40**	**+23**	**+80**
$E\,\left[N/mm^2\right]$	1019,04	7,9	0,52
$\nu\,[-]$	0,41	0,41	0,41

5.3 Boundary conditions and loads

Solar modules are usually built into an aluminum profile. Since the outer layers are made of glass, which is a rather fragile material, direct contact of glass with metal is undesirable, as deformation will result in a high stress concentration, which will lead to the formation of cracks and destruction. Therefore, a special buffer layer is used between the glass and the aluminum frame, which absorbs the difference in deformations and compensates for stress.

Since there is no detailed information about the buffer layer and its functions, it is assumed in the work that all layers of the anti-sandwich can rotate freely at the edges [6, 22]. Therefore, as boundary conditions, it is advisable to choose zero displacements of each layer at the edges in the thickness direction.

To simplify the task, a uniform static load was applied in the form of an anti-sandwich pressure uniformly distributed over the entire plane, orthogonal to the plane of the surface. When choosing the magnitude of the load, it was assumed that the problem is geometrically linear, so the ratio must be fulfilled $w_{max}/h<0{,}5$, where w_{max} – the maximum deflection, h – the total thickness of the anti-sandwich. According to [1], the value of the distributed load can vary within the limits $\pm 2{,}4\cdot 10^{-3}$ and $\pm 5{,}4\cdot 10^{-3}\ N/mm^2$. n this problem, the following load was applied $p = 0{,}5\cdot 10^{-3}\ N/mm^2$.

5.4 Finite element mesh

The finite-element mesh was constructed using spatial shell elements. At the same time, only one element was used for the thickness of the layer. The aspect ratio of each element in planar directions is equal to one: $AR = \frac{h_1^e}{h_2^e} = 1$. Thus, the corresponding nodes on the boundaries between the layers coincide. Since the behavior of the element in the planar directions and in the thickness direction is different, it is important to set the thickness direction, the so-called "stack direction" to the elements during discretization. For this, it is necessary to choose the upper and lower surface. The geometric model with a finite-element mesh is shown in Fig. 3, in which the lower surface is marked in purple.

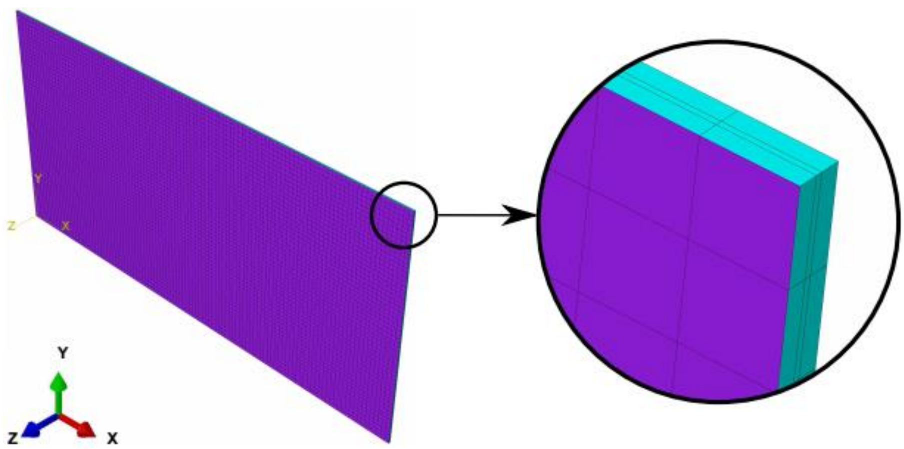


Fig. 3 Spatial discretization of the plate

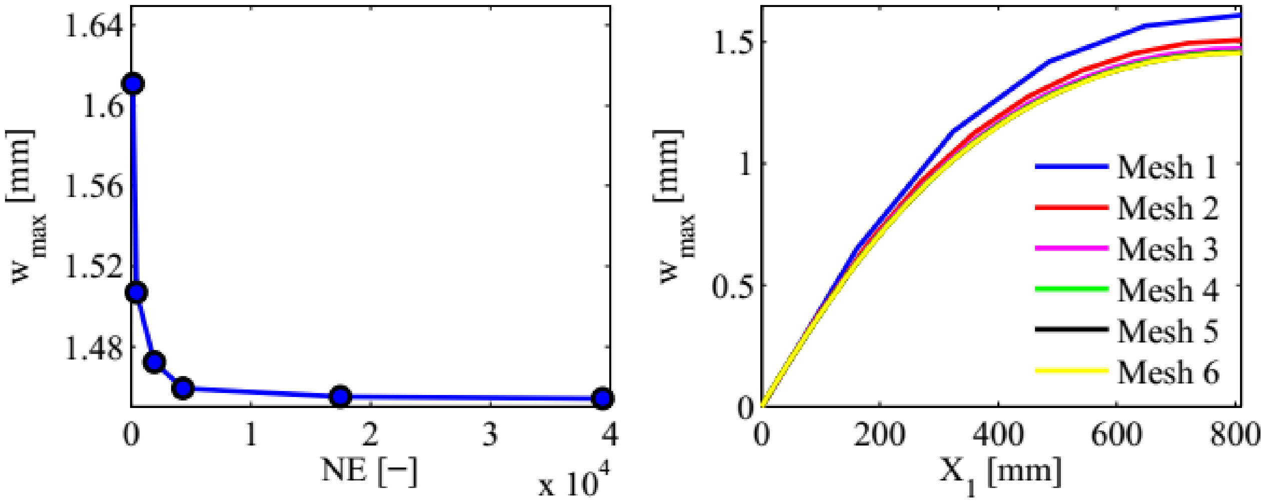
5.5 Study of convergence of results

The study of the convergence of the results was carried out in order to check the accuracy of the finite element calculation. For this, the h-method was used, which consists in increasing the number of elements. In this way, several grids with different numbers of elements were generated, on which the maximum values of plate deflections were calculated and compared with each other. In each mesh, the number of elements increased due to the reduction of their faces, but at the same time the ratio was preserved $AR = 1$.

Since the goal is to determine the mechanical behavior of the anti-sandwich under real weather conditions, research at different temperatures is important.

Numerical simulations were carried out using the ABAQUS [4] software package at the Faculty of Mechanical Engineering at the Otto- von-Guericke University of Magdeburg (Germany).

To investigate the convergence of the results, six finite-element meshes with different numbers of elements were constructed and the values of the maximum deflection of the plate for each mesh at different temperatures were compared. The results are shown in the figures (Fig. 4).



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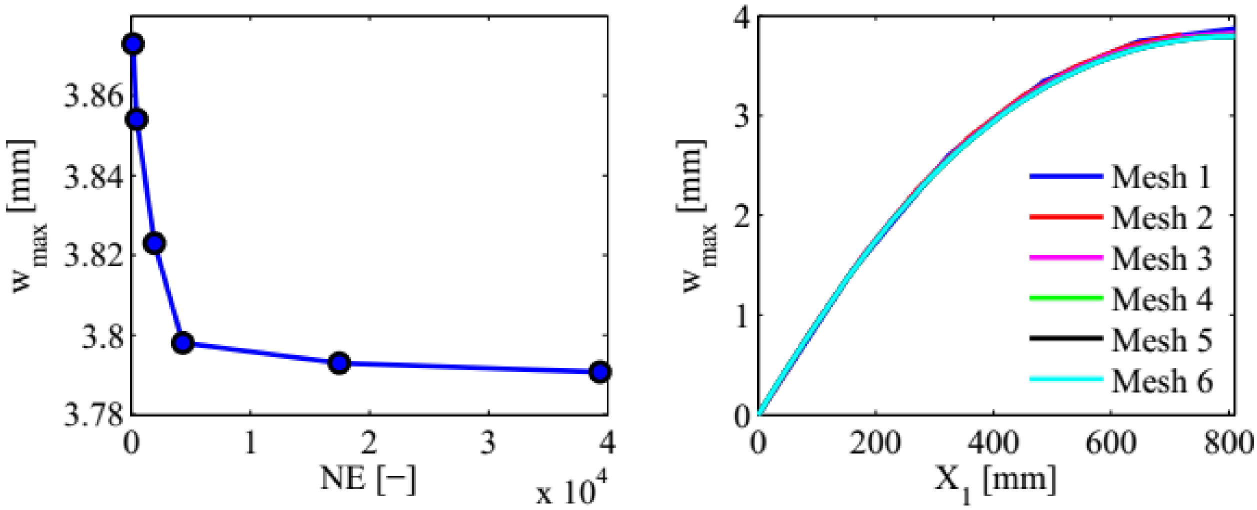

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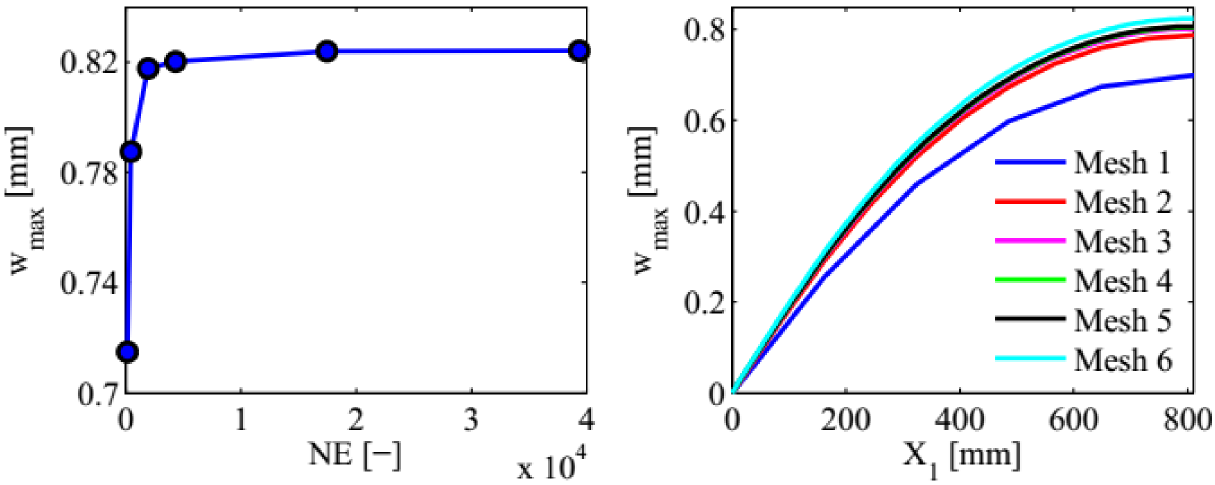

c

Fig. 4 The maximum values of deflections of the plate, calculated using different numbers of elements at different temperatures: a) +23 °C, b) +80 °C, c) -40 °C

The numerical results of the dependence of the maximum deflection of the plate on the number of elements are given in table. 4.

Table 4. The main results of calculations

№	NE	w_{max} (T=+23°C)	w_{max} (T=+80°C)	w_{max} (T=-40°C)
1	150	1,611	3,873	0,7149
2	486	1,507	3,854	0,7874
3	1944	1,472	3,823	0,8176
4	4374	1,459	3,798	0,8201
5	17496	1,455	3,793	0,8238
6	39366	1,454	3,79076	0,82407

From the figures and from the table, it is clear that the deviations between the deflections on the last two grids (#5 and #6) are very small (less than 5%). This means that the solutions coincide, and therefore it is sufficient to use grid 5 for further calculations.

As can be seen from Fig. 4 and Table. 4, the maximum deflection decreases as the number of elements increases. From a physical point of view, this dependence does not correspond to reality, since with more elements, the plate should become less rigid. Accordingly, the deflection should increase.

In contrast, at a temperature of -40 °C, the convergence shows physically correct results, i.e., the deflection increases as the mesh thickens.

5.6 Change in modulus of elasticity

The dependence of the maximum deflection of the anti-sandwich panel on the modulus of elasticity of the middle layer at a given constant load was studied $p = 0{,}5 \cdot 10^{-3}\, N/mm^2$. At the same time, different values of the modulus of elasticity were selected, for each of which the value of the maximum deflection was obtained. As in the previous case, the deflection was determined on the upper surface of the upper layer.

Calculations showed that the increase or decrease of the maximum deflection for different grids depends on the modulus of elasticity of the middle layer. That is, with a higher modulus of elasticity, the deflection of the beam increases with the thickening of the grid. In Fig. 5 shows the results of the convergence study at different elastic moduli of the middle layer.

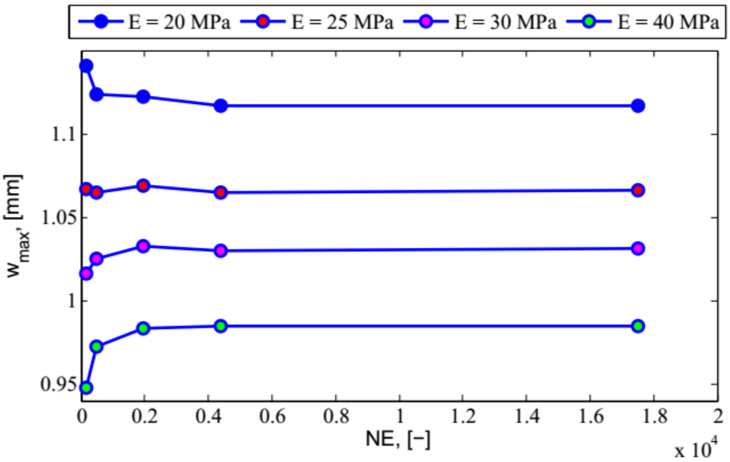

Fig. 5 Convergence of deflections of the plate
at different modulus of elasticity of the middle layer

Thus, a set of points was obtained for which linear interpolation was performed. For clarity, it was decided to take a logarithmic scale for the values of the modulus of elasticity (x-axis on the graph).

Comparing with the available results based on extended layerwise theory (Fig. 6), it can be seen that the displacement values calculated by continuum shell elements [13, 17, 18, 21] are ten times higher than the available ones.

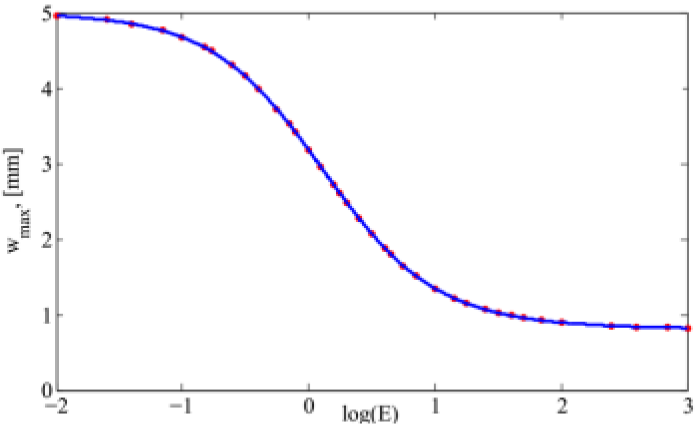

Fig. 6 Dependence of the maximum deflection of the beam on the modulus
of elasticity of the middle layer

The graph shows the logarithmic dependence of the maximum deflection of the plate on the modulus of elasticity of the middle layer. This dependence reflects the real behavior of the solar module at different temperatures. At the same time, different values of the modulus of elasticity correspond to different ambient temperatures. At low temperatures, the modulus of elasticity increases, that is, the middle layer becomes stiffer, and the entire plate is more uniform. Accordingly, the deflection in this case decreases. With a lower modulus of elasticity, that is, at high temperatures, on the contrary, the deflection increases.

Along with temperature loads, solar panels are also subjected to dynamic loads such as wind and precipitation in real conditions. Therefore, the study of the solar panel for oscillations is quite an interesting and important issue. As part of the work, a study of the natural oscillations of the anti-sandwich was carried out, namely, the natural frequencies were determined.

The dependence of natural frequencies on the modulus of elasticity of the middle layer was also investigated. So, for example, Fig. 7 shows a graph of the dependence of the first natural frequency on the change in the modulus of elasticity.

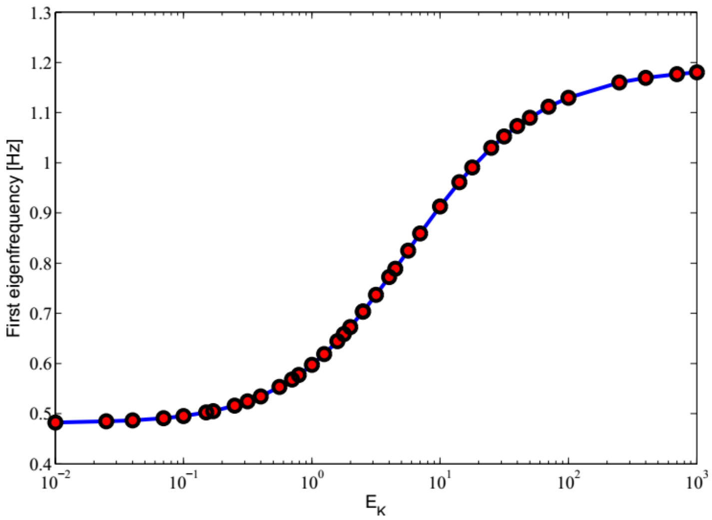

Fig. 7 Dependence of the first natural frequency
on the modulus of elasticity of the middle layer

5. Research results

In order to prove the expediency of using spatial shell elements, a comparative analysis of the results of calculation of deflections, displacements in plane directions and variation of the modulus of elasticity of the middle layer by two methods is given: using spatial shell elements and based on the LWT laminate theory.

Although the above approaches allow the behavior of thin laminate plates with a thin and ductile inner layer to be predicted with high accuracy, the XLWT elements are not standardized. Therefore, the application of these elements is impossible for the general public of engineers. Thus, it is necessary to develop and propose alternative methods of modeling this type of structures. It is appropriate to consider the possibility of using a special type of standardized finite elements built on the basis of the theory of continuum mechanics.

This study analyzes the possibility of applying such elements to the modeling of anti-sandwich panels. It is shown that spatial shell elements (continuum shell elements) (Fig. 8) in the vast majority of cases are not only not inferior in accuracy to rather expensive three-dimensional elements, but also have a sufficiently high efficiency compared to XLWT elements. Together with the fact that these elements are standardized and therefore included in the basic configurations of such a finite element package as ABAQUS, the obtained data lead to a logical conclusion about the high potential of spatial shell elements for modeling the strength and vibration characteristics of solar panels.

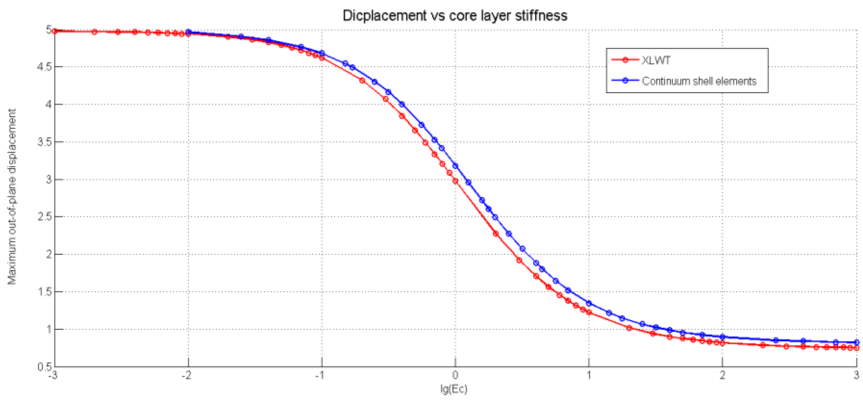

Fig. 8 The maximum deflection of the anti-sandwich panel for different values of the modulus of elasticity of the middle layer when applying spatial shell elements (blue) and the XLWT theory (red)

Fig. 8 shows that the convergence of the results of both theories is quite high. This shows that the model used in this problem adequately describes the behavior of solar cells.

The distribution of deflections in both cases is symmetrical (Fig. 9), the same trend occurs (the deflection increases with increasing temperature). On the one hand, this indicates the correctness of the calculations. However, it should be noted that the values of deflections when using spatial shell elements are 10 times higher than the values of XLWT deflections.

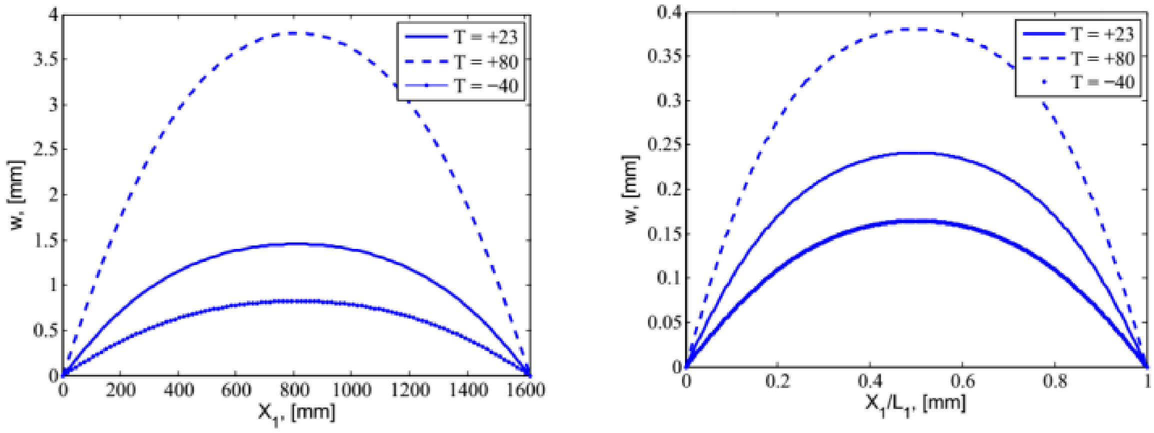

Fig. 9 Plate deflections at different temperatures when using continuum shell elements (left) and extended layerwise theory (right)

Preliminary calculations showed deviations between the results. Therefore, there is a need to check the correctness of the performed calculations with the use of spatial shell elements. Since it is not possible to carry out an exact analytical calculation of the anti-sandwich, it is only possible to carry out calculations in such extreme cases, in which the behavior of the plate can be accurately predicted, i.e. carry out the so-called asymptotic analysis.

The assumption that the properties of the middle layer change depending on the temperature is considered. Thus, we can say that there is such a limiting value of the modulus of elasticity, when the middle layer acquires the same properties as the outer layers ($E_c=E_{зовн}$). In this case, the entire plate becomes homogeneous and shear-rigid, and thus can be calculated according to Kirchhoff's theory for shear-rigid thin uniform plates. The second limiting case is the opposite of this, i.e. it is assumed that the properties of the outer layers coincide with the properties of the inner layer ($E_{зовн}=E_c$). In this case, the plate is uniform and susceptible to shear, and is subject to calculation according to the Mindlin-Reissner theory for soft uniform plates, taking into account transverse shear. So, let's consider two extreme cases in detail: a shear-rigid plate (Kirchhoff theory) and a shear-yielding plate (Mindlin-Reisner theory).

In both cases, a homogeneous isotropic plate is considered, that is, all layers consist of the same material (in this case, glass or EVA).

As part of this analysis, in both cases, the values of displacements in the direction of thickness (deflection) were obtained. The results of the analytical calculations were compared with the results of the corresponding numerical calculations. All equations for analytical calculations were taken from [16].

In the first limiting case, a homogeneous rigid plate was considered. The main assumptions and derivation of the plate equation according to the Kirchhoff theory are given in [16]. The solution of the equation is given in the form of a double Navier row [16]:

$$w(x_1,x_2)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}\sin(\alpha_m x_1)\sin(\beta_n x_2), \qquad \alpha_m=\frac{m\pi}{l_1}, \qquad \beta_n=\frac{n\pi}{l_2} \tag{1}$$

This solution satisfies all boundary conditions in the case of a rectangular isotropic hinged plate not loaded with moments. If we decompose the load q(x1,x2) into a sine double row

$$q(x_1, x_2) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn} \sin(\alpha_m x_1)\sin(\beta_n x_2) \tag{2}$$

where

$$q_{mn} = \frac{4}{l_1 l_2}\int_0^{l_1}\int_0^{l_2} q(x_1, x_2)\sin(\alpha_m x_1)\sin(\beta_n x_2) \tag{3}$$

and substituting into the general solution and equating the coefficients, we obtain the general form of the solution for a rectangular plate [16]:

$$w(x_1, x_2) = \frac{1}{K}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{q_{mn}}{(\alpha_m^2 + \beta_n^2)} \sin(\alpha_m x_1)\sin(\beta_n x_2) \tag{4}$$

With

$$q_{mn} = \frac{16 q_0}{\pi^2 mn}, m, n = 1,3,5... \tag{5}$$

Thus, equation (4) can be used to calculate the first limiting case of the anti-sandwich. At the same time, it is considered that the plate is homogeneous, that is, all three layers are made of glass: $E=E_c=E_в=E_н= 73\cdot10^3 MPa$, $\nu=\nu_c=\nu_в=\nu_н=0{,}3$, $\rho = \rho_c = \rho_в = \rho_н = 25\cdot10^{-7}\, kg/mm^3$; plate thickness $h = h_c = h_в = h_н = 7{,}4\ mm$, planar dimensions $l_1 = 1620\ mm$, $l_2 = 810\ mm$; distributed load $q_0 = p = 0{,}25\cdot10^{-6}\, N/mm^2$.

In this case, the load is taken much less than when calculating the anti-sandwich. This is done in order to fulfill the condition of applying Kirchhoff's theory $w/h \leq 0{,}5$ (because in this case the plate will be much more flexible).

In the second limiting case, a shear-yielding plate was considered, for the calculation of which the non-classical theory of Mindlin-Reissner plates was applied [16].

We get the deflection equation after integration:

$$w = \frac{1}{K}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{q_{mn}}{(\alpha_m^2 + \beta_n^2)^2}\left[1 + \frac{K}{Gh_s}(\alpha_m^2 + \beta_n^2)\right]\sin(\alpha_m x_1)\sin(\beta_n x_2) \tag{6}$$

Taking into account $K = \frac{Eh^3}{12(1-\nu^2)}$ and $G = \frac{E}{2(1+\nu)}$, we have $\frac{K}{Gh_s} = \frac{h^2}{5(1-\nu)}$.

The equations of the Reisner theory were applied in the calculation of the anti-sandwich panel in the second extreme case. At the same time, it is considered that the plate is homogeneous and consists entirely of EVA polymer: $E=E_в=E_н=E_c= 7{,}9\ MPa$, $\nu=\nu_c=\nu_в=\nu_н=0{,}41$, $\rho = \rho_c = \rho_в = \rho_н = 960\cdot10^{-9}\, kg/mm^3$; plate thickness $h = h_c = h_в = h_н = 7{,}4\ mm$, planar dimensions $l_1 = 1620\ mm$, $l_2 = 810\ mm$; distributed load $q_0 = p = 0{,}25\cdot10^{-6}\, N/mm^2$.

As a result of analytical calculations based on the theory of rigid Kirchhoff plates and the theory of flexible Reisner-Mindlin plates, the following maximum values of deflections were obtained:

$w_K = 4{,}2272\cdot10^{-4}$ mm and $w_{R\text{-}M} = 3{,}5722$ mm.

In order to confirm the correctness of the results, along with the analytical solution, a numerical calculation was also carried out using the spatial elements of the shell in two limiting cases. A study of the convergence of the results was also conducted (Fig. 10), the maximum values of deflections on the densest grid (grid 7) were calculated. The parameters of all grids and the corresponding deflection values are given in table 5. In table 6, the corresponding results of the analytical and numerical solution are compared, and the error is also determined.

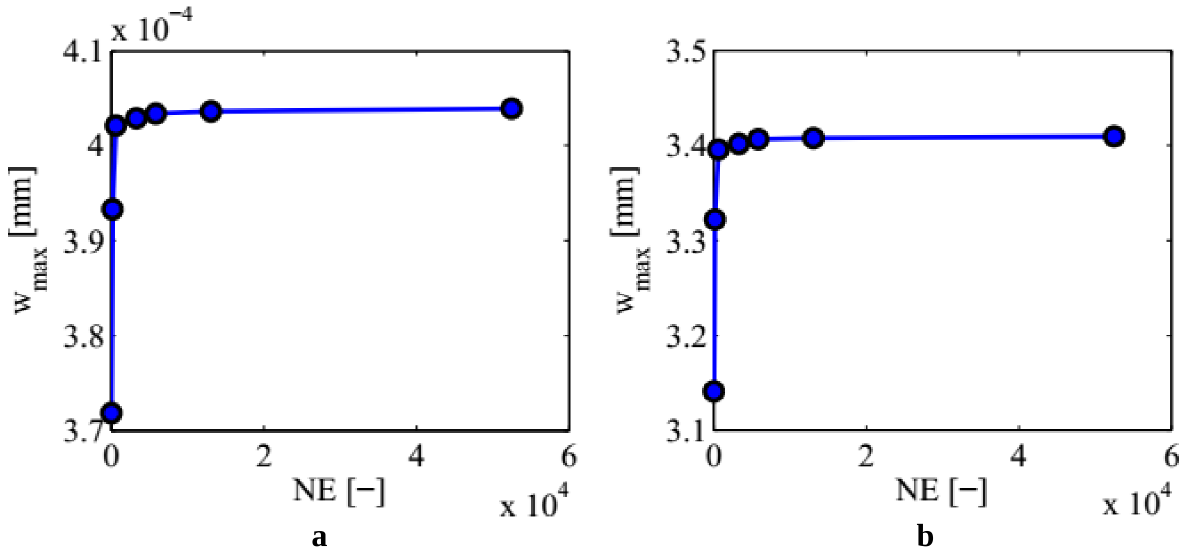

Fig. 10 Study of convergence of results according to Kirchhoff (a) and Reisner (b)

Table 5. Parameters of the mesh into finite elements for the numerical calculation of limit cases

Mesh	NE	Element side length h^e [mm]	$w_K \cdot 10^{-4}$ [mm]	w_R [mm]
1	50	162	3.718	3.141
2	162	90	3.933	3.322
3	648	45	4.021	3.396
4	3321	30	4.029	3.402
5	5832	15	4.034	3.407
6	13122	10	4.036	3.408
7	52488	5	4.039	3.41

Table 6. Results of calculations in limiting cases with static loading of the plate

Parameter	**Kirchhoff**	**Reisner-Mindlin**
w_{an} [mm]	$4,2272 \cdot 10^{-4}$	3,5722
w_{num} [mm]	$4,039 \cdot 10^{-4}$	3,41
δ [%]	4,45	4,54

Table 6 shows that the error between the analysis and the numerical solution for both cases is less than 5%. This suggests that the numerical solution is correct, that is, the spatial shell elements are suitable for the calculation of this design.

For verification, it was decided to calculate the deflections value of the anti-sandwich panel for different values of the modulus of elasticity of the middle layer under load $p = 0,25 \cdot 10^{-6} \, {}^{N}\!/\!{}_{mm^2}$, which corresponds to the load that was applied for the calculation of limit cases. The results are shown in Fig. 11:

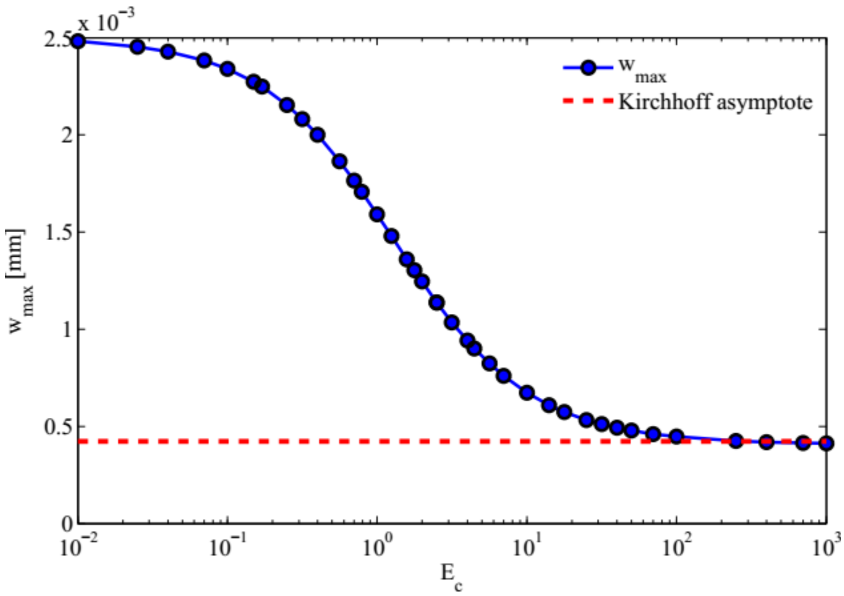

Fig. 11 Maximum values of deflections for different mechanical properties of the middle layer under load $p = 0,25 \cdot 10^{-6}\ N/mm^2$

As can be seen from the figure, the values of the deflections of the anti-sandwich (blue line) lie within the limits of the values for the limit cases, if compared with table 6. Thus, the expediency of using spatial shell elements for modeling the mechanical behavior of the anti-sandwich under various conditions is proven.

As part of the scientific work, the deformed state of a three-layer plate was analyzed using finite element analysis using a spatial shell element. This element is based on the first-order shift theory [4]. In addition, calculations were carried out in the limiting cases of a rigid and shear-yielding thin plate according to the theory of Kirchhoff and Mindlin-Reisner.

As for the theories of multilayer plates, the paper compared the results obtained on the basis of the spatial shell element with the results obtained according to the theory of multilayer plates.

Calculations were carried out according to two theories: Kirchhoff and Reisner-Mindlin theory. Determination of natural frequencies according to both theories was carried out according to [2].

When calculating the natural frequencies according to the first theory, a homogeneous rectangular plate hinged to shear was considered (E_o=E_m=E_l) with dimensions *1620mmx810mmx7,4mm.*

All three layers of the anti-sandwich are rigid and have the mechanical properties of glass:

$E = E_c = E_в = E_н = 73 \cdot 10^3 MPa$, $\nu = \nu_c = \nu_в = \nu_н = 0.3$, $\rho = \rho_c = \rho_в = \rho_н = 25 \cdot 10^{-7}\ kg/mm^3$.

According to Kirchhoff's theory, the first 9 natural frequencies were determined

$$\omega_{mn} = \pi^2 \left(\frac{m^2 + n^2}{l_1^2 + l_2^2} \right) \sqrt{\frac{K}{\rho h}}, m,n = 1,2,3... \quad (7)$$

The results of calculations according to the first theory are given in table. 7.

Table 7. Natural frequencies of the plate (Kirchhoff theory)

№	f_{an}	f_{num}	δ [%]
f_1	1,1457	1,1432	0,22
f_2	1,8332	1,8271	0,33
f_3	2,9789	2,9691	0,33
f_4	3,8956	3,8900	0,14

Continued of **Table 7**

f_5	4,583	4,5692	0,29
f_6	4,583	4,5692	0,29
f_7	5,7288	5,7035	0,44
f_8	6,6454	6,6267	0,28
f_9	7,3328	7,2942	0,53

When using the Reisner-Mindlin theory, the natural frequencies were determined for a shear-yielding hinged homogeneous rectangular plate ($E_o=E_m=E_l$) with dimensions *1620mmx810mmx7,4mm.*

Eigenfrequencies were determined using the following formula

$$\omega_{mn} = \omega_{mn}^{K}\sqrt{\frac{1}{1+\beta_{mn}^{Q\,2}}} \quad , \tag{8}$$

where $\beta_{mn}^{Q\,2} = \frac{K}{Gh_s}\left(\alpha_m^2 + \beta_n^2\right)$ - shift correction factors, ω_{mn}^{K} - the natural frequency of vibration according to the Kirchhoff theory.

All layers are assumed to be made of EVA material: $E=E_в=E_н=E_c=7{,}9MPa$, $\nu=\nu_в=\nu_н=\nu_c=0.41$, $\rho = \rho_c = \rho_в = \rho_н = 960 \cdot 10^{-9}\, {kg}/{mm^3}$.

The obtained values of natural frequencies are shown in the table. 8.

Table 8. Natural frequencies of the plate (Reisner-Mindlin theory)

№	f_{an}	f_{num}	δ [%]
f_1	$2{,}0111 \cdot 10^{-2}$	$2{,}00768 \cdot 10^{-2}$	0,17
f_2	$3{,}2166 \cdot 10^{-2}$	$3{,}20889 \cdot 10^{-2}$	0,24
f_3	$5{,}2245 \cdot 10^{-2}$	$5{,}21436 \cdot 10^{-2}$	0,19
f_4	$6{,}828 \cdot 10^{-2}$	$6{,}8299 \cdot 10^{-2}$	0,03
f_5	$8{,}0303 \cdot 10^{-2}$	$8{,}02345 \cdot 10^{-2}$	0,085
f_6	$8{,}0303 \cdot 10^{-2}$	$8{,}02345 \cdot 10^{-2}$	0,085
f_7	0,1003	0,10016	0,14
f_8	0,1164	0,11635	0,04
f_9	0,1283	0,12809	0,16

In both limit cases, slight errors are observed between the numerical and analytical values (table. 7, 8), which indicates that the shell spatial elements are used correctly and give quite accurate solutions also in the dynamics analysis. This is also evidenced by the fact that the value of the first natural frequency of oscillations, calculated for the anti-sandwich, lies within the limits defined by the limiting cases of Kirchhoff and Reisner-Mindlin theories.

6. Prospects for further research development

In future research related to solar cells, forced oscillations caused by possible dynamic loads, such as precipitation, wind currents, etc., are of considerable interest, and it is reasonable to calculate, along with the natural frequencies, also the natural forms of the oscillations. They carry information about what shape the structure will have if a force is applied to it with a frequency equal to the corresponding natural frequency of oscillation. It is planned to determine the eigenforms of oscillations analytically and numerically with the use of spatial shell elements in two limiting cases: a rigid plate (Kirchhoff theory) and a shear-yielding plate (Mindlin-Reisner theory).

7. Conclusion

The type of finite element suitable for solving this problem is selected.

The study of the solar panel using finite element analysis with the use of spatial elements of the shell is given in the paper. A model (anti-sandwich) was used, which adequately describes the mechanical behavior of the solar panel, which is a three-layer plate with relatively thick and rigid outer layers and a thin and flexible inner layer.

An analytical and numerical calculation of the natural frequencies of vibration in the limit cases of a homogeneous plate was carried out in order to verify the correctness of the numerical calculations under static load conditions. The obtained results have a high correlation, and also showed that the frequencies calculated for the anti-sandwich are within acceptable limits.

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