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Determination of eigen vibration modes for three-layer plates using the example of solar panels

Iaroslav Lavrenko
Department of Dynamics and Strength of Machine and Strength of Materials, National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute», Kyiv, Ukraine
ORCID 0000-0002-4384-4866

Olena Chaikovska
Department of Theory, Practice and Translation of German, National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute», Kyiv, Ukraine
ORCID 0000-0001-9945-4296

Sofiia Yakovlieva
Structural engineer at FAM GmbH, Magdeburg, Germany

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Abstract: Solar panels are considered as three-layer plates with a thick hard outer layer and a thin soft inner layer. To describe the mechanical behavior of the plates on the example of a solar panel, a model for anti-sandwich plates was used. The literature review includes scientific articles describing models for analytical and numerical calculations of three-layer plates. During the scientific study of the mechanical behavior of the solar plate under the influence of external factors, the method of finite element analysis using the element of the spatial shell was used. This type of elements is used for theories of single and multilayer plates. Shell elements were used for calculations and modeling of the natural forms of vibrations of three-layer plates. The paper presents scientific studies under static loading under different exposure conditions, as well as an analysis of self-oscillations of a three-layer plate using the Kirchhoff and Mindlin theories as an example. As part of the scientific work, a study of the mechanical model of a thin solar panel was carried out using finite element analysis in the ABAQUS program, taking into account different temperature conditions. The article provides analytical calculations of the application of various theories to determine the natural forms of plate vibrations.

Keywords: solar panel, anti-sandwich, Kirchhoff theory, Reisner-Mindlin theory, eigen modes, finite element method, ABAQUS.

1. Introduction

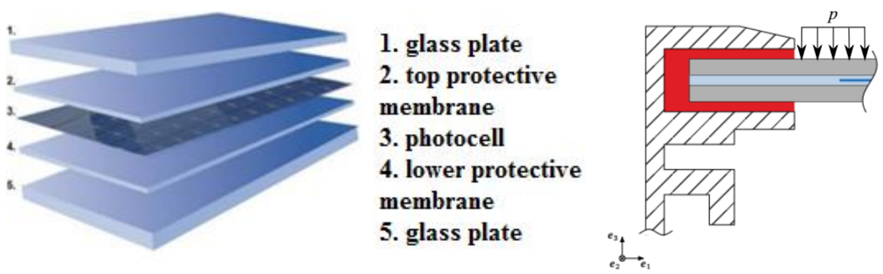
Solar energy is promising and the most unexplored today and in the future. The greatest accumulation of solar energy is possible with the use of solar panels, which are subjected to heavy loads due to changes in temperature, wind and precipitation. The above effects lead to dynamic loads. In this regard, the study of plate vibrations on the example of solar panels is interesting and important for the design of solar panels of greater strength. Therefore, modeling and research of the behavior of individual layers of three-layer plates is a necessary and urgent problem today [1,2].

In scientific works, solar cells are considered as multi-layer plates, the behavior of which is studied by classical theories. Analytical calculations and the results of determining the eigen forms of oscillations for shear-rigid and shear-elastic plates are given in the work. They carry information about what shape the structure will have if a force is applied to it with a frequency equal to the corresponding eigen frequency of vibration.

2. Object and subject of research

The object of the study is an anti-sandwich panel, with the help of which it is possible to describe the behavior of a three-layer plate under the influence of various types of load using the example of solar panels.

A typical structure of solar cells consists of outer glass layers and inner soft polymer layers, which perform a protective function for very thin and fragile silicon solar panels (Fig. 1) [3-5].



a) b)

Fig. 1. a) Structure of a solar panel and b) boundary conditions [2].

As mentioned above, this mechanical model describes a typical structure of solar panels, the structure of which consists of glass and soft polymer materials. This type of structure performs a protective function for very thin and fragile silicon solar panels.

In the scientific articles there is not enough information about the type of solar panel mounting. In this case, the analytical calculation is carried out in two limiting cases of fastening: rigid anchoring and free anchoring, which allows relative displacement of the layers.

Shell elements used in classical theories do not take into account transverse shear, which makes it impossible to model the real behavior of the solar panel in the thickness direction.

3. Target of research

The main aim of the study is to determine the natural form of oscillation of the plate using the classical methods of the theory of oscillations and the finite element method on the example of a solar panel model using spatial elements of the shell.

Study of asymptotic analysis in two limiting cases using the theory of Kirchhoff and Reissner-Mindlin in order to check the correctness of the results.

Determination of the dependence of the structure's eigen modes of oscillations on the change in the mechanical characteristics of the middle layer.

4. Literature analysis

The paper considers a three-layer thin composite, using solar panels as an example. The structure of the solar panel can be considered as a multilayer composite with isotropic properties. Due to specific geometric and mechanical characteristics, such a plate was called an antisandwich [6-8, 14-16]. Thus, the antisandwich is a mechanical model that reflects the real geometry and is able to describe the mechanical behavior of the solar panel. In paper [6], a finite-element analysis of an anti-sandwich panel based on the theory of layer theory, using a specially designed finite element, is given.

Since solar cells consist of multilayer structures, the literature review showed that in works [3, 9, 14-17] three-layer glass beams were studied based on the theory of layered materials.

A new finite element is modeled for modeling multilayer structures [9]. The article shows that there are significant deviations between the results of different theories for a three-layer beam with asymmetric layers. This indicates that the shear theory allows large errors in the calculation of such structures. The results of modeling using the new element are presented in papers [7, 13]. Numerical calculations were compared with analytical results obtained for a three-layer plate with a soft inner layer. Using a new volumetric element, it is possible to automate the process of creating a finite-element mesh of a complex geometric shape [11].

It was established that this method is suitable for calculating multilayer structures with a very thin and soft inner layer. Solar panels have a similar structure. The application of this approach for multilayer plates is reflected in papers [2, 6-8, 10, 13, 20, 21].

In papers [23, 24, 25, 26], the results of calculations and modeling of the behavior of sandwich plates of different shapes and made of different materials using the theory of shear deformation are presented. The results of experimental studies of three-layer plates are also presented [27].

5. Research methods

Theories of single and multilayer plates

During the research, an analysis of the deformed state of the three-layer plate was carried out by the method of finite element analysis using the spatial shell element. This element is based on the first-order shift theory [4]. The article presents calculations in the limit cases of a rigid and shear-elastic thin plate according to the theory of Kirchhoff and Mindlin-Reisner.

A comparative analysis of the results obtained on the basis of the spatial shell element with the results obtained according to the theory of multilayer plates was also carried out.

The main equations and assumptions of all used theories are presented later in the paper.

Kirchhoff's theory of rigid plates

A plate is a solid body, one of whose dimensions (thickness h) is significantly smaller than the other two (Fig. 2).

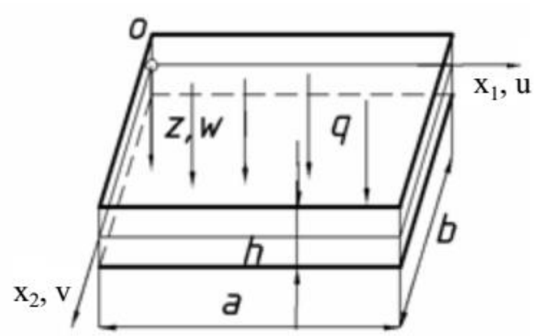


Fig. 1 Schematic representation of a single-layer plate [17].

In the general classification, plates are divided into thin and thick. In thin plates, the thickness is 10-20 times smaller than the planar dimensions. If the ratio of the thickness to the smaller of the planar dimensions does not exceed 1/3, then such a plate is considered thick. The plane parallel to the x_1Ox_2 plane, which divides the plate in half, is called the middle plane [17].

If the displacement of the plate along the z axis (deflection) lies within $w \leq 0,5h$, then such a plate can be calculated according to the classical theory of thin plates (Kirchhoff theory). In this case, along with the main assumptions of the theory of elasticity, additional assumptions are fulfilled [18]:

– The plate material is homogeneous and isotropic

– The middle plane of the plate during bending deformation is a neutral surface, i.e. higher-order deformations and stresses during plate deformation are neglected. Accordingly, the following displacements are obtained: $u_1(x_1,x_2,0)=u_2(x_1, x_2,0)=0, u_3(x_1,x_2,0)=w\neq 0$.

All points on the normal to the undeformed median plane after deformation remain points normal to the deformed median plane. The distance between the points of the normal does not change after deformation, that is, the plate does not stretch in the direction of the thickness $\varepsilon_{33}=0$. Equation of movements:

$$\begin{aligned} u_1\left(x_1,x_2,x_3\right) &= x_3\psi_1\left(x_1,x_2\right) \\ u_2\left(x_1,x_2,x_3\right) &= x_3\psi_2\left(x_1,x_2\right) \\ u_3\left(x_1,x_2,x_3\right) &= w\left(x_1,x_2\right) \end{aligned} \tag{1}$$

– where ψ_1 and ψ_2 – angles of rotation of the cross-section of the plate relative to the axis x_2 and x_1.

The normal stress σ_{33} is generally much smaller than the normal stresses σ_{11} and σ_{22}, i.e. $\sigma_{33} << \sigma_{11}, \sigma_{22}$. Therefore, it is assumed that $\sigma_{33}\approx 0$ (plane stress state).

To derive the equation of the plate, it is necessary to formulate the kinematic relationships between stresses and strains, set the equations of equilibrium (kinetics) to determine the forces and moments acting on the plate, and introduce the relationship between stresses and strains (equations describing the behavior of the material). In the process of deriving this theory, [18] was used.

Kinematic equations

Fig. 3 shows a plate made of a homogeneous isotropic material in an undeformed state (a). Consider the element (b) cut out of it. In the initial state, all surfaces of the element are straight and parallel to each other. After deformation, the element acquires some curvature, but due to the kinematic hypothesis $\varepsilon_{33}=0$, the distance between parallel planes does not change. At the same time, in this problem, a flat deformed state is assumed, i.e. $\gamma_{13}=\gamma_{23}=0$. Taking into account the assumption of a neutral surface, there are no displacements u_1, u_2 of the middle surface, but they are different from zero in the planes parallel to it.

In this case, the following equations are fulfilled:

$$\begin{aligned} &\cos\varphi_1 \approx \cos\varphi_2 \approx 1 \\ &\sin\varphi_1 \approx \varphi_1 \approx \tan\varphi_1 = w_{,1} \\ &\sin\varphi_2 \approx \varphi_2 \approx \tan\varphi_2 = w_{,2} \end{aligned} \tag{2}$$

Thus, the equations for displacements have the form:

$$\begin{aligned} u_1\left(x_1,x_2,x_3\right) &= -x_3 w_{,1}\left(x_1,x_2\right) \\ u_2\left(x_1,x_2,x_3\right) &= -x_3 w_{,2}\left(x_1,x_2\right) \\ w\left(x_1,x_2,x_3\right) &= w\left(x_1,x_2\right) \end{aligned} \tag{3}$$

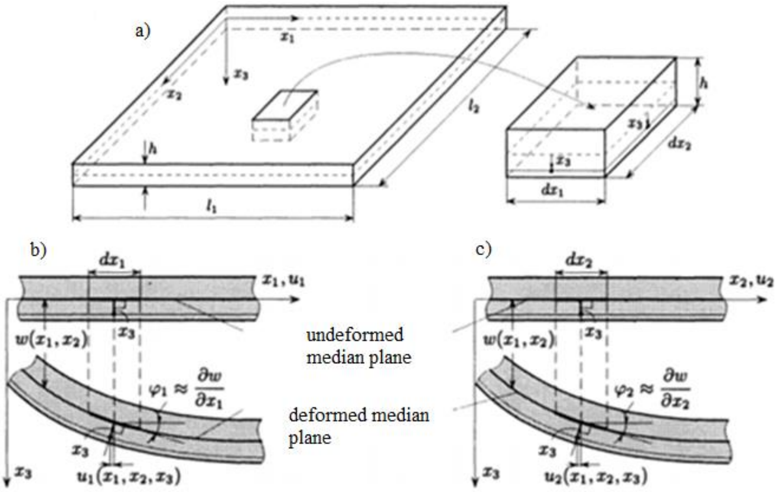


Fig. 2 Kinematic relations in the plate element: a) a rectangular plate and an element cut from it (initial state); b) section in the x_1-x_3 plane of the deformed and undeformed element; c) section in the x_2-x_3 plane of a deformed and undeformed element [18].

Deformations obtained taking into account the relationship between displacements will take the form:

$$\begin{aligned}&\varepsilon_{11} = u_{1,1} = -x_3 w_{,11} = x_3 \kappa_{11},\ \ \varepsilon_{22} = u_{2,2} = -x_3 w_{,22} = x_3 \kappa_{22}\\&\gamma_{12} = u_{1,2} + u_{2,1} = -2x3w_{,12} = 2x_3\kappa_{12},\ \ \gamma_{21} = u_{2,1} + u_{1,2} = -2x3w_{,21} = 2x_3\kappa_{21}\end{aligned} \quad (4)$$

Equilibrium equation

All stresses in the plate by the method of integration over the thickness are reduced to forces and moments related to the length (Fig. 4):

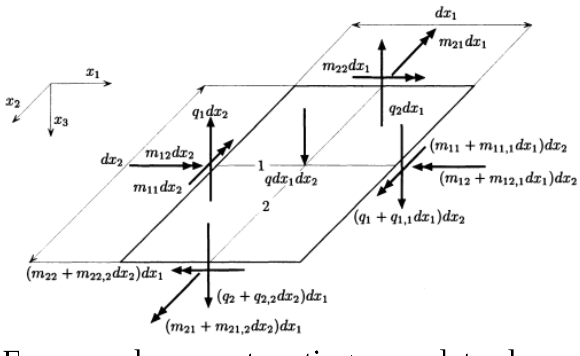


Fig. 3 Forces and moments acting on a plate element [18]

The equilibrium equation is formulated as the sum of all forces and moments acting on the undeformed element of the plate. Neglecting higher orders of differentiation, the equilibrium equation can be reduced to the following system:

$$q_{1,1} + q_{2,2} + q = 0,\ \ m_{12,1} + m_{22,2} - q = 0,\ \ m_{11,1} + m_{21,2} - q = 0 \quad (5)$$

After differentiation, we get the following

$$m_{11,11} + 2m_{12,12} + m_{22,22} = -q \quad (6)$$

The relations between stresses and strains (Hooke's law)

$$\varepsilon_{11}=\frac{1}{E}(\sigma_{11}-\nu\sigma_{22}),\ \varepsilon_{22}=\frac{1}{E}(\sigma_{22}-\nu\sigma_{11}),\ \gamma_{12}=\frac{2(1+\nu)}{E}\sigma_{12}$$
$$\sigma_{11}=\frac{E}{1-\nu^2}(\varepsilon_{11}+\nu\varepsilon_{22}),\ \sigma_{22}=\frac{E}{1-\nu^2}(\varepsilon_{22}+\nu\varepsilon_{11}),\ \sigma_{12}=\frac{E}{2(1+\nu)}\gamma_{12} \qquad (7)$$

After substituting kinematic equations, equilibrium equations and performing mathematical transformations, we obtain the following expression:

$$\frac{\partial^2}{\partial x_1^2}\left[K\left(w_{,11}+\nu w_{,22}\right)\right]+2\frac{\partial^2}{\partial x_1\partial x_2}\left[(1-\nu)Kw_{,12}\right]+\frac{\partial^2}{\partial x_2^2}\left[K\left(w_{,22}+\nu w_{,11}\right)\right]=q \qquad (8)$$

where $K=\frac{Eh^3}{12(1-\nu^2)}$ - bending stiffness the plate. Thus, taking into account the homogeneity of the plate (K=const), equation (3) is reduced to the form:

$$\Delta\Delta w\left(x_1,x_2\right)=\frac{q\left(x_1,x_2\right)}{K} \qquad (9)$$

Kirchhoff's finite element. The points of the middle surface during bending of the plate change their coordinates in the direction of the z axis. Lines that are perpendicular to the median surface remain straight and perpendicular after deformation, as shown in Fig. 5. Thus, it is considered that there is no shift. A point that does not lie on the middle surface has displacement *u* and *v* in *x* and *y* coordinates, respectively. In figure $w_{,x}$ and $w_{,y}$ small angular displacements after deformation of the plate. So

$$u=-zw_{,x}\,,\ v=-zw_{,y}$$

Then
$$\varepsilon_x=u_{,x}=-zw_{,xx}\,,\ \varepsilon_y=v_{,y}=-zw_{,yy}\,,\ \gamma_{xy}=v_{,x}+u_{,y}=-2zw_{,xy} \qquad (10)$$

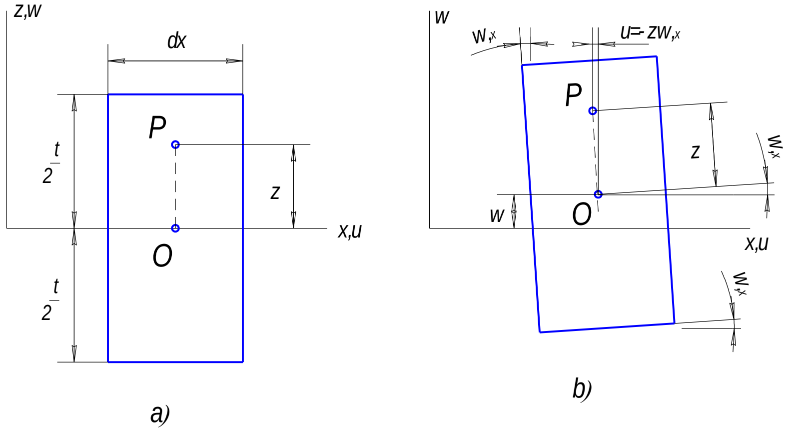


Fig. 5 a) Finite element of thin platinum before loading, b) After loading: the deformations are related to the Kirchhoff theory for plates. Point P is shifted by a coordinate w up and by an angle $zw_{,x}$ to the left, because the middle line has a linear w and small angular $zw_{,x}$ displacement.

Theory of plates with consideration of shear

The classical theory of plates, given in the previous section, is performed under the condition of a thin plate and small displacements $w\le 0{,}5h$. In the case of plates of medium thickness ($0.1\le h/a<0.3$), as well as non-fulfillment of the last condition, i.e. $w>0{,}5h$, non-classical theories of plates that take into account shear deformations are used. One of these theories is the theory of first-order shear deformations, which is also called the Mindlin theory, which describes a shear-yielding plate with small deflections [18].

Basic assumptions:

– the displacement and rotation of the median plane due to deformation along the thickness are not taken into account: $u_1(x_1, x_2, x_3 = 0) = 0; u_2(x_1, x_2, x_3 = 0) = 0,$

– $u_3(x_1, x_2, x_3 = 0) = 0,$

– the deflection of the plate is small compared to the thickness: $w/h \le 0,5$. As a result, the curvature is linearized: $\kappa_{11} \approx \psi_{1,1}; \kappa_{22} \approx \psi_{2,2}; \kappa 12 \approx \psi_{1,2} + \psi_{1,2}$

– the points normal to the middle surface do not change the distance between each other due to deformation: $\varepsilon_{33}=0$. After deformation, they lie on a straight line that is not perpendicular to the median plane.

– the normal stress σ_{33} in shear-yielding plates is neglected.

Kinematic equations

To determine the kinematic ratios, the plate element is considered (Fig. 6).

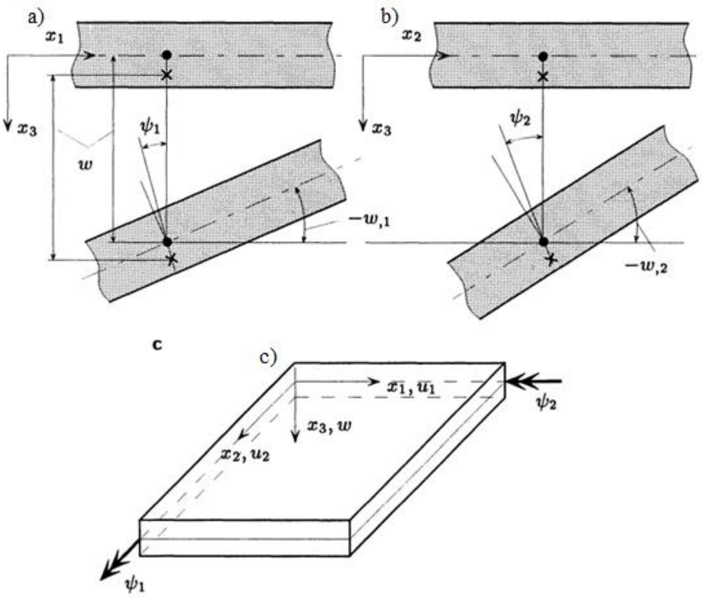


Fig. 6 Kinematics of a shear-yielding plate: a) section x_2=*const*; b) section x_1=*const*; c) movement and angles of rotation of the plate [18]

$$u_1(x_1, x_2, 0) = 0,\ u_2(x_1, x_2, 0) = 0,\ u_3(x_1, x_2, 0) = w \ne 0 \tag{11}$$

In this case, we have:

$$\begin{aligned} &\varepsilon_{11} = u_{1,1} = x_3\psi_{1,1},\ \varepsilon_{22} = u_{2,2} = x_3\psi_{2,2},\ \varepsilon_{33} = 0 \\ &\gamma_{12} = u_{1,2} + u_{2,1} = x_3\left(\psi_{1,2} + \psi_{2,1}\right),\ \gamma_{31} = u_{3,1} + u_{1,3} = w_{,1} + \psi_1,\ \gamma_{23} = u_{2,3} + u_{3,2} = \psi_2 + w_{,2} \end{aligned} \tag{12}$$

In contrast to Kirchhoff's theory, the deformations depend not only on the deflection w, but also on the angles of rotation ψ_1 and ψ_2.

Equilibrium equation (5), (6) for Kirchhoff's theory, they also apply in the case of a plate susceptible to shear.

The relations between stresses and strains:

$$\begin{gathered}\varepsilon_{11}=\frac{1}{E}\left(\sigma_{11}-\nu\sigma_{22}\right),\ \varepsilon_{22}=\frac{1}{E}\left(\sigma_{22}-\nu\sigma_{11}\right)\\ \gamma_{12}=\frac{1}{G}\sigma_{12}=\frac{2(1+\nu)}{E}\sigma_{12},\ \gamma_{23}=\frac{1}{G}\sigma_{23}=\frac{2(1+\nu)}{E}\sigma_{23},\ \gamma_{31}=\frac{1}{G}\sigma_{31}=\frac{2(1+\nu)}{E}\sigma_{31}\\ \sigma_{11}=\frac{E}{1-\nu^2}\left(\varepsilon_{11}+\nu\varepsilon_{22}\right),\ \sigma_{22}=\frac{E}{1-\nu^2}\left(\varepsilon_{22}+\nu\varepsilon_{11}\right)\\ \sigma_{12}=G\gamma_{12}=\frac{E}{2(1+\nu)}\gamma_{12},\ \sigma_{23}=G\gamma_{23}=\frac{E}{2(1+\nu)}\gamma_{23},\ \sigma_{31}=G\gamma_{31}=\frac{E}{2(1+\nu)}\gamma_{31}\end{gathered}\tag{13}$$

Taking into account the equations of kinematics, equilibrium equations and introducing replacement $\psi_{1,1}+\nu\psi_{2,2}=\Phi\left(x_1,x_2\right)$, $\psi_{2,1}-\psi_{1,2}=\Psi\left(x_1,x_2\right)$, receive:

$$\begin{gathered}K\left[\psi_{1,111}+\nu\psi_{2,112}+(1-\nu)\left(\psi_{2,112}+\psi_{1,122}\right)+\psi_{2,222}+\nu\psi_{1,122}\right]=-q\\ w_{,1}=-\psi_1+\frac{K}{Gh_s}\left(\Phi_{,1}-\frac{1-\nu}{2}\Psi_{,2}\right),\ w_{,2}=-\psi_2+\frac{K}{Gh_s}\left(\Phi_{,2}-\frac{1-\nu}{2}\Psi_{,1}\right)\end{gathered}\tag{14}$$

In equations (14), the real thickness h is replaced by the reduced shear thickness h_s. It is obtained through the shear correction factor χ, which in the case of an isotropic plate of level 5/6: $h_s=\chi h=h/1,2$.

In the first equation of system (8), terms with ν are shortened. For the second and third equations, we consider that $w_{,11}+w_{,22}=\Delta w$. In this case, we formulate the equation of the plate:

$$K\Delta\Phi=-q,\ \ \Delta w=-\Phi+\frac{K}{Gh_s}\Delta\Phi,\ \ \frac{1-\nu}{2}\frac{K}{Gh_s}\Delta\Psi-\Psi=0\tag{15}$$

Mindlin's finite element. In this element, it is assumed that the lines that are perpendicular to the median surface in the initial state change their position. Shear deformations are assumed to be present. The displacement of a point that does not lie on the middle surface is not described by the derivatives of displacements $w_{,x}$ and $w_{,y}$ as in Kirchhoff's theory. When using the Mindlin element, the point movement depends on the angles θ_x and θ_y [2]. That $u=-z\theta_x$, $v=-z\theta_y$

Then $\varepsilon_x=-z\theta_{x,x}$, $\varepsilon_y=-z\theta_{y,y}$, $\gamma_{xy}=-z\left(\theta_{x,y}+\theta_{y,x}\right)$, $\gamma_{yz}=w_{,y}-\theta_y$, $\gamma_{zx}=w_{,x}-\theta_x$ (16)

The above formulas follow from the ratios

$$\varepsilon_x=u_{,x},\ \varepsilon_y=v_{,y},\ \varepsilon_z=w_{,z}\ \ \gamma_{xy}=v_{,x}+u_{,y},\ \gamma_{yz}=u_{,z}+w_{,y},\ \gamma_{zx}=u_{,z}+w_{,x}.$$

Mindlin's finite element takes shear into account, so it is possible to analyze thick and layered plates Fig. 7.

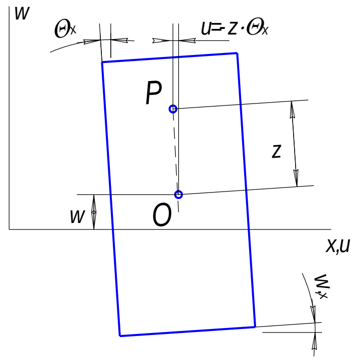


Fig. 7 The differential element of the plates after deformation, similar to Fig. 5, but taking into account the shift ($w_{,x} \neq \theta_{,x}$ therefore $\gamma_{zx} = w_{,x} - \theta_x \neq 0$)

For a rectangular plate, the shape functions N_i can be expressed in *x* and *y* coordinates, and the area element is $dA = dxdy$. If the element is curvilinear, as shown in Fig. 8, then the shape functions N_i can be expressed in isoparametric coordinates ξ and η [3].

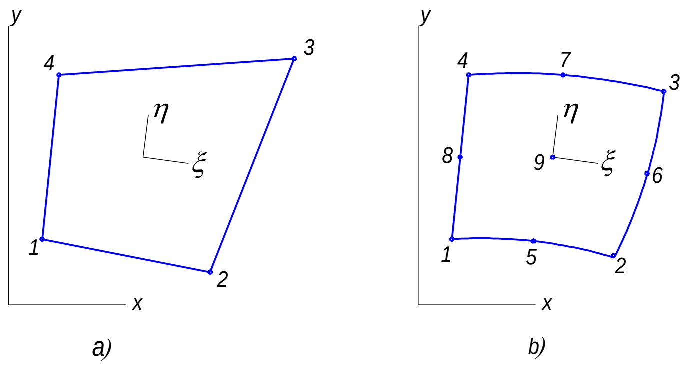


Fig. 8 a) Bilinear element and b) quadratic element.

Accordingly, for bilinear and quadratic elements, the shape functions N_i are given in the form [4]

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta) \qquad N_2 = \frac{1}{4}(1+\xi)(1-\eta) \quad N_3 = \frac{1}{4}(1+\xi)(1+\eta) \quad N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

and tables Fig. 9.

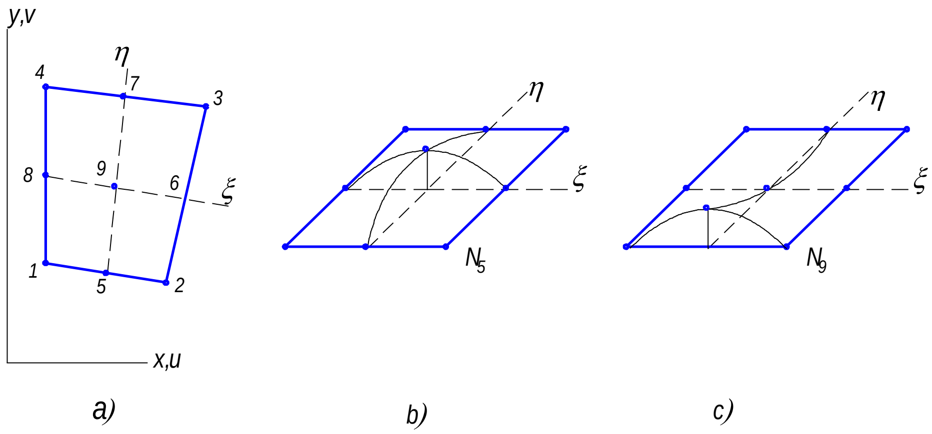


Fig. 9 a) Element with nine nodes in Cartesian coordinates.
b),c) Functions of the form N_9 and N_5, for quadratic elements in $\xi - \eta$ coordinates.

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta) - \frac{1}{2}(N_5 + N_8) - \frac{1}{4}N_9 \qquad N_2 = \frac{1}{4}(1+\xi)(1-\eta) - \frac{1}{2}(N_5 + N_6) - \frac{1}{4}N_9$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta) - \frac{1}{2}(N_6 + N_7) - \frac{1}{4}N_9 \quad N_4 = \frac{1}{4}(1-\xi)(1+\eta) - \frac{1}{2}(N_7 + N_8) - \frac{1}{4}N_9 \quad (17)$$

$$N_5 = \frac{1}{2}(1-\xi^2)(1-\eta) - \frac{1}{2}N_9 \quad N_6 = \frac{1}{2}(1+\xi)(1-\eta^2) - \frac{1}{2}N_9 \quad N_7 = \frac{1}{2}(1-\xi^2)(1+\eta) - \frac{1}{2}N_9$$

$$N_8 = \frac{1}{2}(1-\xi)(1-\eta^2) - \frac{1}{2}N_9 \qquad N_9 = (1-\xi^2)(1-\eta^2)$$

6. Research results

Determination of eigen vibration forms of oscillations using Kirchhoff and Mindlin elements was carried out by means of numerical modeling using the ABAQUS software package at the Otto-von-Guericke Institute of Mechanics of the University of Magdeburg (Germany) using spatial shell elements.

The analysis of natural oscillations was carried out according to the two theories of Kirchhoff and Mindlin-Reissner in the case of a rigid homogeneous plate ($E_в=E_н=E_c$) and a soft homogeneous plate ($E_c=E_в=E_н$) [1]. The equations for determining the natural frequencies and natural forms were taken from the source [2,3,4].

Fig. 10 and Fig. 11 show the natural forms of oscillations, calculated analytically and numerically using spatial shell elements in two extreme cases: a rigid plate (Kirchhoff theory) and a shear-yielding plate (Mindlin-Reisner theory). From the figures, it is clearly seen that the eigenmodes coincide in both limit cases during analytical and numerical calculations, which, in turn, proves that the calculation with spatial elements of the shell gives accurate results.

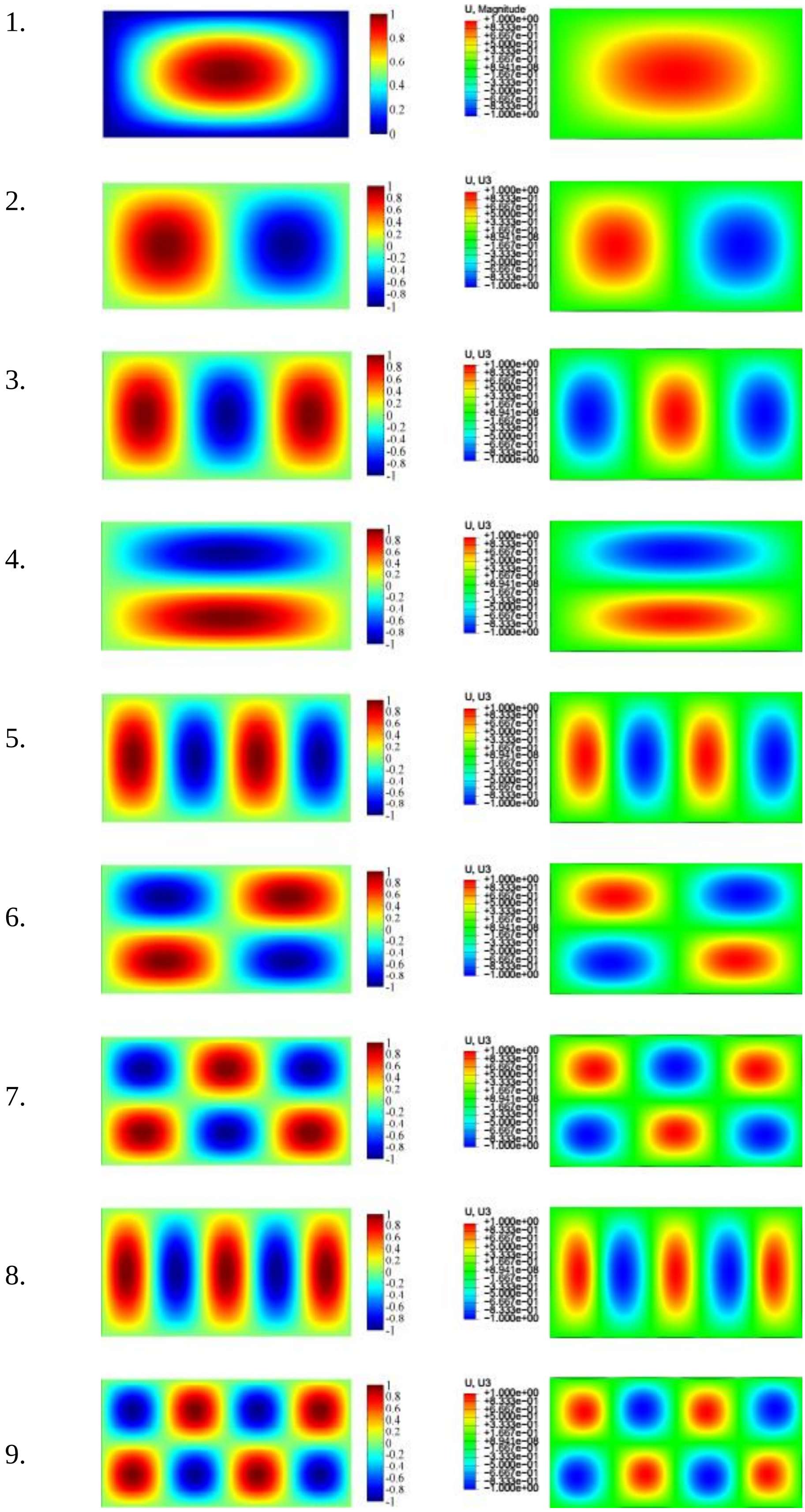


Fig. 10 Eigenmodes of vibrations in the limiting case of a shear-rigid plate, calculated analytically (left) and with the use of spatial shell elements (right)

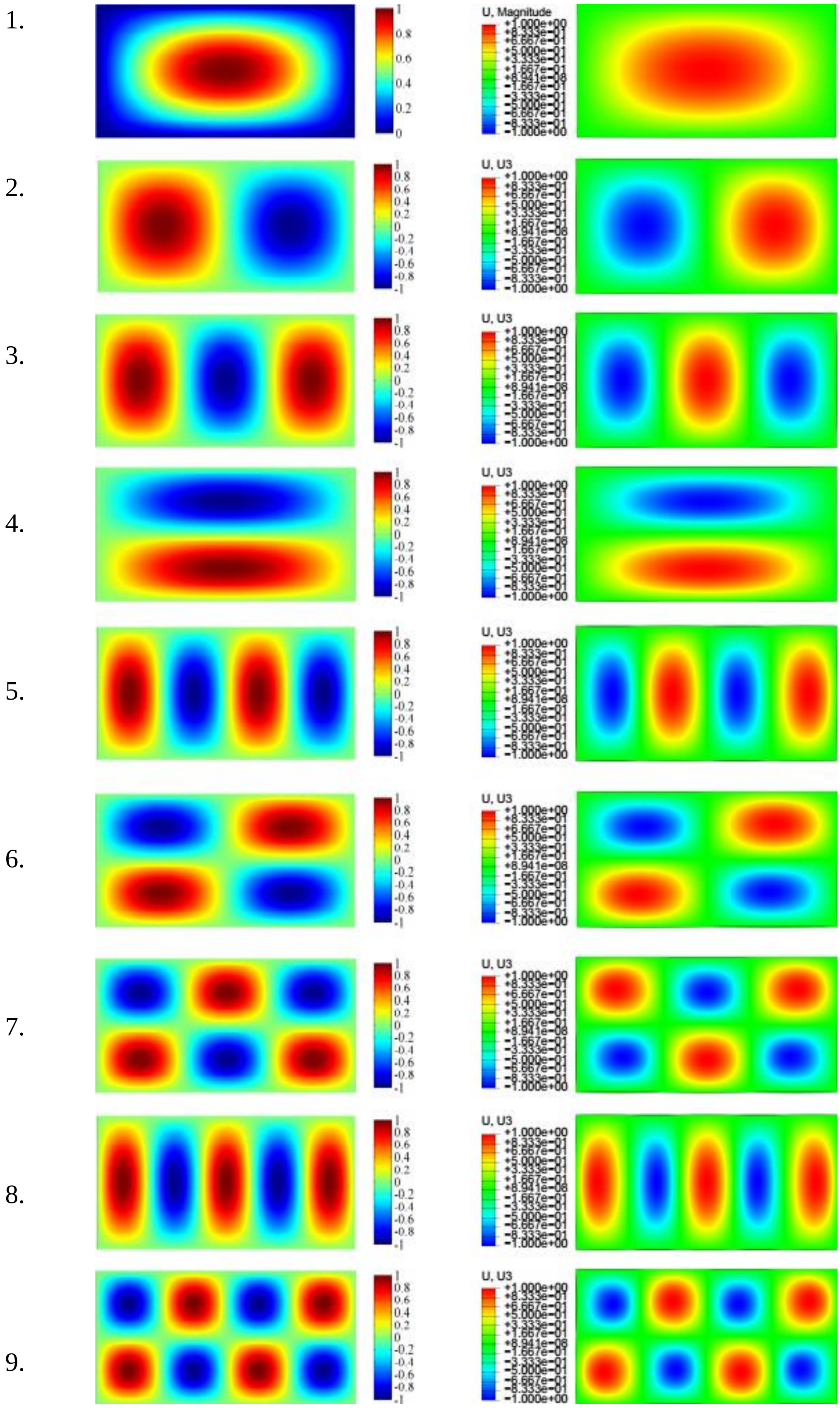


Fig. 11 Eigenmodes of vibrations in the limiting case of a plate susceptible to shear, calculated analytically (left) and with the use of spatial shell elements (right)

7. Prospects for further research development

In future research related to multilayer plates, forced oscillations caused by loads of various types, as well as natural forms of oscillation of structures with complex geometric shapes, are of considerable interest. It is planned to determine the above-mentioned characteristics using the Kirchhoff and Mindlin theories.

8. Conclusion

The paper presents the results of the study of the behavior of a multilayer plate, using the example of a solar panel, by the method of finite element analysis using spatial elements of the shell. An anti-sandwich plate model was used to describe the mechanical behavior of the solar panel, which is a three-layer plate in structure.

To check the adequacy of the analytical calculations, a numerical calculation of the eigenforms in the limit cases of a homogeneous plate was carried out. The obtained results showed a high convergence of numerical and analytical analysis.

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