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Time horizon role consideration in assets “prudent” and “cap” extremes estimation of stochastically evolving value

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Abstract: Paper is focused on analysis and assessment of time horizon or project duration role in property upper or “cap” and lower or “prudent” value estimation. Based on stochastic modelling of market value evolution over time with its roots in Samuelson’s Rational Theory of Warrant Pricing methodology of assessing the role of project duration on these value extremes is proposed. Results of model testing provided using Ukrainian and British residential property market data bases demonstrated its soundness and general efficiency. Summarized results for different residential property types including houses, apartments and land plots in this respect are presented. In response to new international banking regulations set by Basel 3.1 Accord and implemented recently in requirements of European Regulation CRR3 generalized model for estimation of property “cap” and “prudent” value is described. The model presented in multi-factor regression type equations for estimation of adjustment parameters to be applied to property market value stochastically evolving over time for adequate estimation either “cap” or “prudent” value. Main parameters included into the model account for market general tendency, its volatility level, time horizon needed and level of confidence required. Generalized form of the model presented opens an opportunity of its broader implementation for other markets and types of assets.

Keywords: Stochastic model, value volatility, prudent value, cap value, time horizon, adjustment parameters, model testing, value estimation

1. Introduction and overview

After two-years postponement of implementation caused by the COVID-19 pandemic new deadline dated 01 January 2025 has been set by the EU governing bodies to implement amendments provided by updated version of Capital Requirements Regulation (CRR) commonly known as CRR3 [1]. New scope of EU requirements for credit and investment institutions transpose set of international banking supervisory rules finalized under the Basel III Framework or shortly Basel 3.1 Accord with extended requirements that have been approved by the Basel Committee of Banking Supervision (BCBS) in 2017 and include now revised prudently conservative valuation criteria.

Consequently, all jurisdictions within EU are required to implement new regulation from the very beginning of 2025 to provide a new step in strengthening financial system, its stability and ability to mitigate challenges of external and internal origin. With this revised version of CRR3 regulation concept of prudential value applied for banking collateral lending purposes now embraces also immovable assets including commercial and residential real estate property.

Through the analysis of key roots and outcomes of Global Financial Crisis 2007-2009 real estate lending was identified as a major contributing factor and consequently needs for more robust and reliable valuation as essential part of mortgage lending origination and monitoring process was determined.

To underline the scale of real estate market it could be mentioned that its global size is estimated to reach an impressive value of US$654.40 trillion by 2025 [2]. That is by 5.8 times exceeds the level of worldwide GDP expected for the same year [3]. As a normal practice real estate is widely used for collateral purposes in banking lending operations. At the same time generalized data from US, Europe and Japan indicate that a half of the fall in the value of banks during Global Financial Crisis and COVID-19 effect was caused by the level of falls in Real Estate Investment Trust prices [*Kohlscheen and Takats,* 2020: 4]. It coincided with the European Systematic Risk Board statement that "evidence shows that one of the main causes of past banking and financial crises has been credit-driven real estate "boom/bust" cycles"[*ESRB*, 2022: 5].

This might be taken as a practical prove of regulatory decisions taken by the BCBS being now implemented in Europe on immovable property prudential valuation for secured lending purposes. It confirms also important role of real estate lending in the financial system that became especially obvious during and after last GFC 2007-2009 with unproven over-lending as a main crisis cause.

At the same time it's well recognized that there is currently no agreed interpretation of the definition and proposed valuation methodology for providing a Prudential Value in a real estate context [6]. In contrast to fair-valued financial instruments with prudent valuation specified by the EBA Regulatory Technical Standard [7] there is almost no guidance yet from a real estate perspective for the determination of this new 'Prudential Property Value' from the BCBS, nor from European authorities.

Basically, appearance of "prudent value" concept and definition is linked with assets value fluctuations over time or time effect. Since most widely used concept of "market value" provides estimation of appropriate value at fixed period of time for any other point of time during the life of the loan or any other type of financial project property value might be lower or higher of such estimated level. For lending purposes conservative approach to provide necessary adjustments to assets market value in general is accepted being stipulated by the amended European Regulation 575/2013 [1].

To enhance further resilience of financial system it's globally recognized that market value solely is not sufficient and necessary conservative adjustments are needed to exclude any possible over-lending in banking operations. This basic concept is now reflected in updated EU prudential requirements which are still grounded on assets market value to be adjusted accordingly. It means that assets market value as a concept and key basis of value still should play a core role in prudential valuation paradigm. This seems entirely appropriate, since to estimate the necessary adjustments we need to capture the expected time-related changes in the market value of assets recorded at a given point in time.

With property market value evolving over time stochastically under the influence of whole spectrum of acting exogenous and endogenous factors for practical purposes two main time related parameters should be considered when estimating level of such adjustments to market value needed. These are market evolution general trend and level of process evolution stochasticity. Splitting analysis of assets market value by these two main components will help not only to clarify the role of each of them but also to simplify the overall analysis with regard to time effect when analytically estimating assets prudentially conservative value depending on time horizon needed.

This could be considered as a main focus of the manuscript oriented to present some results of analytical approach developed to estimate both low or "prudent" value bound and its upper or "cap" counterpart with particular attention to the time horizon needed and testing of the results achieved using market evolution data sets available.

2. Review of existing approaches

Main content of Basel Committee on Banking Supervision (BCBS) new requirements for the valuation of real estate for lending purposes is included in so-called 'Standardised Credit Risk Assessment Approach' (SCRA). It states that "… to ensure that the value of the property is appraised in a prudently conservative manner, the valuation must exclude expectations of price increases and must be adjusted to take into account the potential for the current market price to be significantly above the value that would be sustainable over the life of the loan."[8].

From this basic statement at least three main guidelines could be taken. First, prudential valuation should reflect lowest level of property market value during the loan time horizon with necessary adjustment when needed. Second, property market value should be used as the basis for such adjustment. Finally, loan duration should be also considered when estimating magnitude of adjustment to be provided.

When transposing these Basel 3.1 requirements into new EU regulation more structures requirements have been included. Article 229 of CRR 3, in particular, states that:

- for immovable property collateral, the collateral shall be valued at or at less than the market value … or at or less than the mortgage lending value;
- valuation should not to take into account speculative elements in the assessment of the market or mortgage lending value;
- the value of the collateral shall be the market value or mortgage lending value reduced as appropriate.

These guidelines and requirements should be taken as key reference points when discussing current status of approaches available for estimating property prudent value and developing any further proposals in this sense.

Main necessity to correct property market value for collateral lending purposes is related to value stochastic evolution over time. Based on that key approaches proposed could be called under the generic heading of long-term value determination.

Historically probably most well known and recognized by regulatory bodies concept of value time effect consideration is Mortgage Lending Value (MLV) initially appeared in Germany beginning of this century [*Grimman*, 2017: 9]. Being mainly driven by the Association of German Mortgage Banks (*German Pfandbrief Banks-VDP)* in a close cooperation with the Germany Federal Financial Supervisory Office MLV concept is well established and came into force in 2005 through the adoption of the German Pfandbrief Act that regulates the determination of the mortgage lending value.

From that time on MLV concept plays a central role in property valuation for collateral lending purposes in Germany and to some extent in several other European countries like Austria, Czechia, Hungary, Luxemburg, Poland, Slovenia and Spain [RICS, 2018: 10]. At the same time principle based framework of MLV model led to perceptible distinctions in its application in these jurisdictions and hence doesn't support principles of uniformity.

In general sense estimation of the property MLV is based on current transaction databases available that should be accompanied by a broader analysis from the past as well as a reasonable forecast analysis to be added to arrive to the "prudent" sustainable value. Based on this approach is grounded on combined consideration of available historic empirical data, current market values and future-oriented assessments that ignore short-term price volatility to arrive to a realistic lowest possible level of market value, which is expected in a future time horizon under consideration.

Quite a different view on time effect consideration in property valuation was proposed by Nordlund [Nordlund, 2008:11] and Cardozo [Cardozo et al, 2017:12] that have been named as "reference value model" and "adjusted market value-AMV", respectively. Both of them are also grounded on the previous statistical data through the identification of long-term trends based on past data available, assuming that the future would look like the past. The main distinguishing feature of these approaches from the MLV-based type is the smoothing of local volatility in property market value by "averaging" the overall trend, rather than focusing on estimating long-term conservative value.

More recent version of AMV-approach is grounded on comparing asset's current market value with long-term trend line as reflected in an appropriate capital value index evolution. The regression-generated, long-term trend line is drawn dynamically rather than with historical hindsight through an inflation-adjusted capital value index [*Cardozo et al*, 2017:12].

Further advancements of such models include some consideration of the trend evolution with forecast of its pattern in coming years. This forecast is mainly based on conventional discounted cash flow (DCF) techniques [*Burston, Burrel*, 2015:12, *Crosby, Hughes*, 2011:14].

Comparison of long-term value estimation main approaches performed by the Property Industry Alliance working group gave preference to AVM methodology as most reliable in reflecting cyclical market behavior [*PIA*, 2017: 15]. At the same time lack of robustness of all methodologies tested has been underlined when applied at the real estate industry specific sectors levels.

With respect to cyclical mode of value change evolution two main concepts of time effect consideration have been identified in summarizing report prepared by working group set up by leading international valuer's organizations [*Crosby, Hordijk*, 2021: 16]. Stylized presentation of these two main concepts of long-term value estimation is given by fig.1.

The first one is "through-the-cycle" type which averages or "flattens" the value pattern fluctuations through the time period, aiming to identify a fair economic or equilibrium value. The main application of such models covers mostly investment type of financial analyses and decisions with time effect concern.

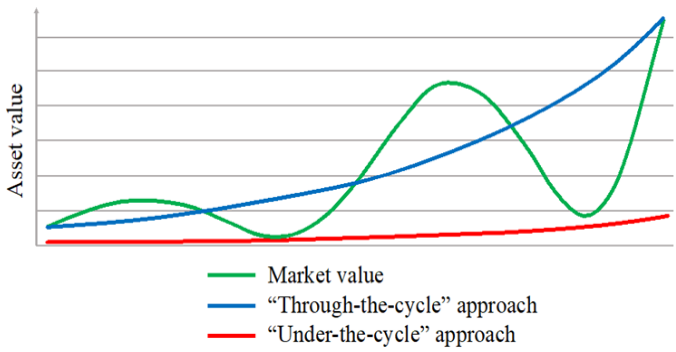


Fig. 1. Stylized presentation of "through-the-cycle" and "under-the-cycle" approaches of long-term value estimation.

Source: Crosby N., et al, 2021: 16.

It's quite evident that long-term value methodologies that fall into this "through-the-cycle" concept are not in compliance with both Basel 3.1 and CRR 3 key prudent value notion that it should not be higher than market value. It will be the case for all lower part of market cycles evolution.

The second one is "under-the-cycle" concept which intends to find lower "skirting" line of market value fluctuation over a given period of time. This second type of methodologies are more in line with established Basel 3.1 and CRR3 requirements to implement prudent assessment of assets value with respect to a long-term effect. As stated in these regulations assets prudent or long-term value should not exceed the market value at any time under consideration.

At the same time based on main constraint related to the fact that "it is virtually impossible to construct a robust, consistent prudent valuation regime at the level of the individual property" research recommends a combination of market value supplemented by a through-the-cycle long-term value to provide market adjustment. Same position was further argued and developed in the next stage research report published by Crosby N. and Hordijk A. [Crosby, et al, 2023: 17].

With the certain individual particulars in place one of the main commonalities of all long-term "prudent" value estimation methodologies based both on "through-the-cycle" and "under-the-cycle" concepts are their schematization of value evolution pattern over time as demonstrated by fig.1 when assets market or fair value should be corrected to reach its long-term "prudent" value by deduction of certain adjustments. Such adjustments are not analytically and statistically grounded at individual property level being mainly based on real property segments value general trend over time.

Same approach is enforced in more structured approach of prudent value estimation by EBA Regulatory Technical Standard (*EBA*, 2020: 18) applicable to financial instruments with deduction of Additional Valuation Adjustments (AVAs) from the Common Equity Tier One (CET1) capital.

This regulatory document allows application of two approaches to prudent valuation. The simplified approach, applicable for financial institutions with total absolute fair-valued assets and liabilities below EUR 15 bln prescribes total AVA equal to 0.1% of the total fair value. The core approach, compulsory for institutions above the EUR 15 bln threshold, prescribes the calculation of 9 AVAs, referring to most influential sources of valuation uncertainty to achieve the prudent value with 90% level of confidence. To take into account correlation between different uncertainties considered aggregation factor of 66 % is provided when arriving to AVAs total level [*EBA*, 2020: 18].

More analytically grounded model of assessing time effect in valuation and estimating long-term "prudent" value was proposed being successfully tested based on Ukrainian residential property data base [*Yakubovsky, et al*, 2022: 19]. The model has been generalized further on with embracement estimation not only lower or "prudent" value" but also its extreme counterpart – upper or "cap" value [*Yakubovsky et al*, 2024: 20]. That reflects to some extent the need of developments in this area that was strongly underlined by the international group of experts representing most recognized international valuation professional bodies [*Crosby and Hordijk*, 2021: 16].

With regard to the nature and character of value evolution over time developed methodology is grounded on the theory of stochastic processes for solving this practically needed task of reliable estimation of both lower bound or "prudent" and upper bound or "cap" assets value. In this sense the problem of estimation extremes of stochastically progressing value was represented as an analytical task of finding upper or lower border of this randomly evolving process that will not be reached with given level of probability during the period of time requested.

As an analytical description of property market value stochastic evolution basic elements of Samuelson's Rational Theory of Warrant Pricing was utilized [*Samuelson*, 1965: 21]. Being well-known and recognized as a foundation of stochastic financial mathematics this theory has found wide spread applications in different important branches of post-neoclassical economics [*Merton*, 2006: 22].

At the same time analytical decisions on diffusion type stochastic processes approach to upper and lower boundaries are usually focused on estimation of probability distribution function of the first-time approach to certain boundary or first two moment of that parameter, i.e. mean time and dispersion [*Sveshnikov*, 2007:23]. It creates a certain complication because we need to solve opposite task being interested to find out not a probabilistic parameters of first-time boundary approach but rather magnitude of the boundary for Markov process evolving over time horizon subject of analysis.

Such analytical solutions for assets market value V_t stochastically evolving over time t adjustment parameters $\Delta_{t,p,cap}$ for the upper or "cap" bound and $\Delta_{t,p,prud}$ for lower or "prudent" value with the given level of probability p are given respectively in [*Yakubovsky, et al*, 2022: 19; *Yakubovsky et al*, 2024: 20] being presented in the following form:

$$\Delta_{t,p,cap} = V_0\,(\exp^{y_p} - 1); \quad \Delta_{t,p,prud} = V_0\,(1 - \exp^{x_p}), \tag{1}$$

where V_0 is the asset value at the process analysis starting point when $t=0$, when notions x_p and y_p stand for parameters that reflect stochastic behavior of property market value evolution. These parameters subject of determination by a simple dichotomy approach based on market value evolution data available with its further evolution forecast. Successful testing of the model both for upper and lower bounds extremes estimation has been provided in relation to Ukrainian residential property market including apartments, houses and land plots.

Schematic view of $\Delta_{t,p,cap}$, $\Delta_{t,p,prud}$ adjustment parameters involvement in a process of "cap" and "prudent" value estimation is given in fig.2 for growing, stagnating and falling markets general trend.

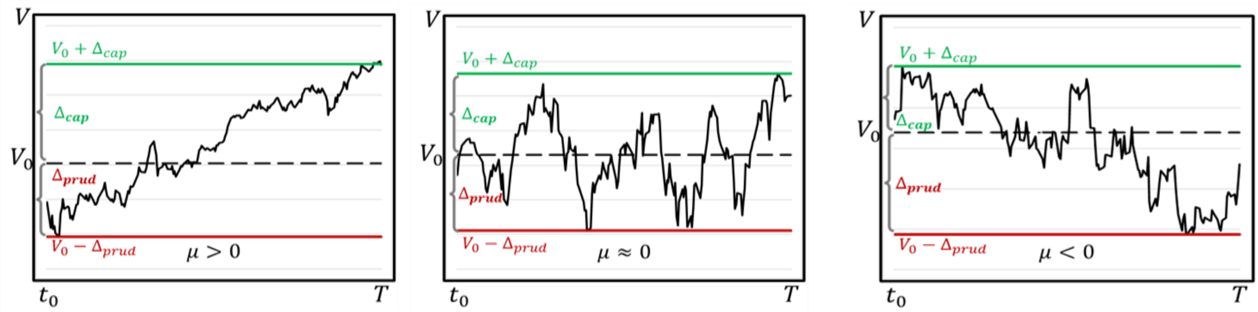


Fig.2. Schematic presentation of long-term "caps" and "prudent" value estimation for growing (μ>0), stagnating (μ~0) and falling (μ<0) markets.

Source: Yakubovsky et al, 2024: 20.

With reference to adjustments parameters $\Delta_{t,p,cap}$, $\Delta_{t,p,prud}$ calculated by (1) with required level of reliability level p should be applied to correct property market value V_0 at initial point of time t_0 to arrive at upper or "cap" property value by

$$V_{t,p,cap} = V_0 + \Delta_{t,p,cap}; \tag{2a}$$

or lower - "prudent" property value by

$$V_{t,p,prud} = V_0 - \Delta_{t,p,cap} \tag{2b}$$

Such methodological scheme when property market value is used as a reference is in line with general approach for prudential valuation adjustments included in Basel 3.1 and CRR3 regulations. In its core it also satisfies main requirements postulated in these regulatory documents in relation to property market value hence "prudent' value estimated within developed approach with given level of probability/certainty will never exceeds market value.

Among most important advantage of that approach is its solid theoretical foundation based on Samuelson's seminal manuscript widely recognized and successfully utilized in different branches of stochastic financial mathematics. For practical application as above it gives a solid ground and flexibility for future application when time effect and other considerations in stochastic markets analysis are needed.

At the same time previous 2 publications devoted to development and verification of this approach which was named as "follow-the-cycle" type didn't consider in particular influence of time horizon on level of adjustment parameters. Quite evident even from the general scheme given in fig.2 that especially for growing and falling markets influence of time horizon might be essential. Apart from this essential point generalization of results achieved with their further comparative verification conducted using British residential property market are presented below.

3. Methodology snapshot

Using notation proposed by *Shiryaev* et al [23] stochastic evolution of the asset value V_t within Samuelson’s model of “diffusion” type can be described by the following equation:

$$V_t=V_0 \ exp\{(\mu-\sigma^2/2)t+\sigma W_t \}, \qquad t \geq 0. \tag{3}$$

Here, $V_0 > 0$ is asset value at $t = 0$ point of time being nonrandom and known, $\mu \in R$ is process growth factor, $\sigma > 0$ is process volatility, and W_t is a Wiener process with continuous paths.

Essentially assets stochastic evolution in accordance with (3) depends on three key parameters: process “randomness” determined by coefficient of volatility through dispersion σ, general tendency coefficient μ and process duration t.

Actually rewriting equation (4) in terms of log-values we obtain:

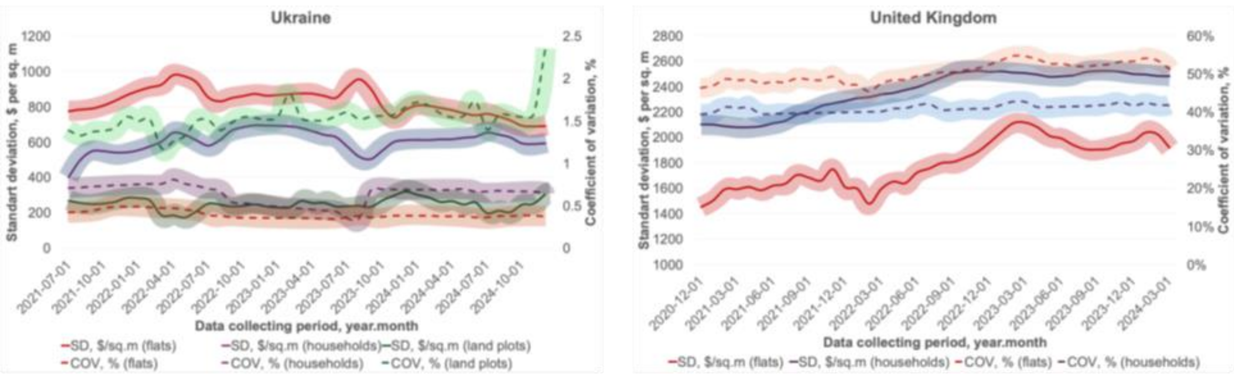
$$V_t=V_0+at+\sigma W_t, \qquad t \geq 0 \tag{4}$$

where: $a= \mu - \sigma^2/2, \qquad t \geq 0$.

In more general form equation (4) can be rewritten as:

$$V_t = V_0 + \Delta_{d,p} + \Delta_{s,p} = V_0 + \Delta_{t,p} . \tag{5}$$

Such interpretation and split of general correction parameter $\Delta_{t,p}$ into 2 main components, i.e. deterministic and stochastic, that depend mainly from process duration $\Delta_{d,p}$ and its volatility $\Delta_{s,p}$ can be accepted for stationary stochastic processes with $\sigma W_t \simeq$ Const [*Tikhonov, Mironov*, 1977:24]. This assumption is also proven by results of volatility evolution characterized by dispersion and coefficient of variance for different types of residential property as presented in fig. 3 for Ukrainian and British markets.



a) Ukraine. *b) UK*

Fig 3. Standard Deviation and Coefficient of Variation of Real Estate Prices in Ukraine (a) and the UK (b) (2020–2024).

Source: author’s own creation based on data from [24, 25].

This study utilizes an initial datasets of the UK and Ukrainian residential property markets, which are continuously monitored and updated by the University College London and by consulting-engineering group of companies “VERITEX”, respectfully, under the authors' methodological and operational guidance as for Ukraine. These primary databases are constructed by monitoring and accumulating information flows through gathering data from various sources within the existing real estate market followed by in-depth statistical processing by analyzing the collected data to extract meaningful insights.

The data collection period varies by property type and region, ranging from 43 to 66 months. This period encompasses two COVID-19 pandemic outbreaks happened in spring 2020 and autumn 2021 and the ongoing war in Ukraine that commenced in February 2022. The size of the market in terms of the number of property units is substantial especially for flats which dominate the datasets in both Ukraine and the UK. The total number of property units exceeds 1 million for Ukraine and 2 million for UK.

Ukraine's real estate market exhibits greater price instability with land plots and flats showing significant volatility. The UK market appears more stable than Ukraine's but still shows cyclical volatility likely reflecting economic conditions, housing policies, and demand-supply dynamics. Notably, it shows a steady rise in price dispersion, with households experiencing higher fluctuations than flats.

Applying this approximation to property upper or "cap" value bound $\Delta_{t,p,cap}$ and low or "prudent" value bound $\Delta_{t,p,prud}$ with both adjustment parameters determined by (1), we can write down accordingly that:

$$\Delta_{t,p,cap} = \Delta_{d,p,cap} + \Delta_{s,p,cap} \text{ and } \Delta_{t,p,prud} = \Delta_{d,p,prud} + \Delta_{s,p,prud} \qquad (6)$$

Since the level of the overall adjustments $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ are determined, it is possible now to define its deterministic and stochastic components $\Delta_{s,p,cap}$, $\Delta_{s,p,prud}$, which depend primarily on the degree of variability/volatility of the process and does not significantly depend on process duration *t*, given the empirically confirmed nearly stationarity of the property value random process evolution (fig.3). This could be done by subtracting the time-dependent components $\Delta_{d,p,cap,}$ $\Delta_{d,p,prud}$ from the obtained total adjustment parameters $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$.

Thus, by dissecting the overall adjustments for the upper and lower bounds of assets value stochastic process evolution over time, we can estimate time-dependent components $\Delta_{d,p,cap,}$ $\Delta_{d,p,prud}$ necessary for the required value adjustments. As a final step dependence of total adjustments $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ on process duration *t* could be directly estimated.

Practical application of modelling described above has been done using 2 data sets of residential property market evolution. The first one is information-analytical data base of Ukrainian residential market evolution, created and fed on a constant basis by the consulting-engineering group of companies "VERITEX®". Creation of that primary database is carried out by monitoring and accumulating information flows from the existing real estate market and their subsequent in-depth statistical processing [*Veritex*, 2024: 24].

Time range covered by this primary data base of residential apartments used for the assessment of the parameters was 54 months from July 2019 till December 2024. Three main caterogies of real estate property have been covered that includes apartments, householdings and land plots. More detailed description of this data base is given in [*Yakubovsky, et al*, 2022: 19].

Second one is extracted from British residential property market evolution data base being constantly supported by the University College of London [*UCL*, 2024: 25]. For proper comparison of results comparison Great Britain initial data base was processed based on the same methodology as Ukrainian one. Statistical analysis of the total amount of available primary information was performed after its initial filtering based on the Romanovsky's criterion for exclusion of statistical "outliers".

Using one of the most statistically powerful Pearson's χ^2 criterion most appropriate theoretical distribution law for the data set at certain period of time has been analyzed. Repeated calculations performed same as for Ukrainian residential property market data set demonstrated that closest theoretical distribution for British market as demonstrated by fig.4 is lognormal one for the value of 1 square meter of living space.

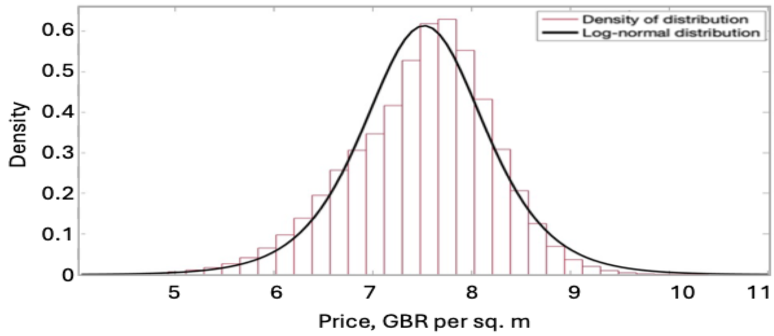


Fig 4. Description of the density of distribution property prices in the market of the UK as of May 2024 by the log-normal distribution law.
Source: author’s own creation based on data from [25].

This statistically proven conclusion methodologically is quite important keeping in mind that it coincides with Samuelson’s Rational Theory main assumptions and also it gives the right to make further initial British data set processing same way as Ukrainian one.

Main parameters needed in our practical cases both for British and Ukrainian markets property different types have been determined as follows. Based on datasets available with time intervals T=1 month following steps to estimate main parameters μ and σ have been done using log-returns:

$$q_k := \ln\left(\frac{U_{k\tau}}{U_{(k-1)\tau}}\right), k = 1, \dots, n \tag{7}$$

The log-returns q_k follow a distribution expressed as:

$$\frac{q_k}{\sqrt{\tau}} = a\sqrt{\tau} + \sigma\gamma_k, k = 1, \dots, n, \tag{8}$$

where $\gamma_k = \tau^{-\frac{1}{2}}(B_{k\tau} - B_{(k-1)\tau}), k \geq 1$, are independent standard normal variables.

For the model of observations (7), the maximum likelihood estimator of a is given by:

$$\hat{a} = \frac{1}{n\tau}\sum_{i=1}^{n} q_i = \frac{1}{n\tau}\ln\left(\frac{U_{T_0}}{U_0}\right) \tag{9}$$

and the unbiased estimator of σ is computed as:

$$\hat{\sigma}^2 = \frac{1}{(n-1)\tau}\sum_{i=1}^{n}(q_i - \tau\hat{a})^2 \ . \tag{10}$$

From this growth parameter μ is determined as:

$$\hat{\mu} = \hat{a} + \frac{\hat{\sigma}^2}{2} \tag{11}$$

In the ensuing steps the maximum likelihood estimators $\hat{a}$ and $\hat{\sigma}$ for a, σ, respectively, are substituted within the given expressions. It is imperative to underscore that the methodology for deriving the estimators x_p and y_p is intricately detailed in [*Yakubovsky, et al*, 2022: 19].

This nuanced and systematic approach distinguishes the methodologies employed for each estimator. Subsequently, the culmination of this methodology yields the final correction parameters for both the upper or “cap” bound ($\Delta_{t,p,cap}$) and the lower or “prudent” property value bound ($\Delta_{t,p,prud}$).

To find out direct dependence of these adjustments from time horizon or project duration needed methodological steps described above and summarised in (5), (6) have been followed.

4. Results and Discussion

Basic results necessary for value adjustments $\Delta_{d,p,cap,}$, $\Delta_{d,p,prud}$ calculation and their time horizon dependence for British and Ukrainian residential property market for the period 2019-2024 are demonstrated in Table 1. For proper comparison same time period was taken for British and Ukrainian residential property markets initial data. It varies by property type ranging from 43 to 66 months depending on initial data availability for Ukrainian market. In general this period encompasses two COVID-19 pandemic outbreaks happened in spring 2020 and autumn 2021 and ongoing war in Ukraine commenced in February 2022.

Table 1. Key parameters for estimation of correction adjustments $\Delta_{d,p,cap}$, $\Delta_{d,p,prud}$ for different residential property type in the UK (flats, houses) and Ukraine (flats, houses, land) as of 2019-2024

Region	Property type	Period, months	Property units, amount	Median Value, $/sq.m	Disper-sion σ^2	Variation coef-t υ	Para-meter μ
Ukraine	Flats	65	884316	980.39	0.0070	0.18	0.008
	Houses	43	159830	500.00	0.1147	0.35	-0.100
	Land	43	36578	442.86*	0.0177	0.15	-0.003
UK	Flats	66	1926900	3626.50	0.0068	0.08	0.009
	Houses	43	284973	5938.47	0.0032	0.05	0.012

** - in $/100 sq.m.*

The overall size of the market in terms of the number of property units is substantial, especially for flats which dominate the datasets both in the Ukraine and the UK. In Ukraine flats and houses have a relatively low median value in comparison with them at British market that is quite explainable by different level of economy in these countries.

At the same time statistical spread of the market data characterized by dispersion and coefficient of variation is remarkably higher for Ukrainian residential property market. General market trend for last several years through its indicator μ in case of Ukraine shows close to zero growth for flats with negative results for houses and plots of land being simply explained and caused by ongoing hostilities. In the UK, flats and houses have much higher median values compared to Ukraine, with lower variances and coefficients of variation, indicating more stable property values and positive growth rates.

Based on these key results for periods of time indicated in table 1for different property types, selected specific probability levels $p_1 = 0.6827, p_2 = 0.9545$, $p_3 = 0.9973$ and various expected market growth parameters μ adjustments parameters $\Delta_{d,p,cap}$, $\Delta_{d,p,prud}$ was calculated with starting time point $t = 0$ at December 1, 2024 in all cases. Several comparisons of calculation results gained and demonstrated in tables 2, 3 for Ukrainian and British residential property market accordingly can be made.

Tables 2, 3 demonstrate level of adjustment parameters $\Delta_{d,p,cap}$, $\Delta_{d,p,prud}$ in absolute values which are quite comparable for same level of reliability p needed and market growth rate μ expected for Ukrainian and British residential property markets. Strong dependence of adjustments needed to get either “cap” or “prudent” property value on reliability level needed and overall market tendency expected. With transition from falling market characterized by negative parameter μ to stagnating and further to growing market tendency with positive μ-level upper adjustment parameter $\Delta_{d,p,cap}$ is increasing. Opposite behavior with parameter μ growth demonstrates “prudent” value adjustment parameter $\Delta_{t,p,prud}$. Such overall performance is in line with schematic overview demonstrated by fig. 2.

Table 2. Adjustment parameters $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ level in USD/sq. m. for residential property value in Ukraine

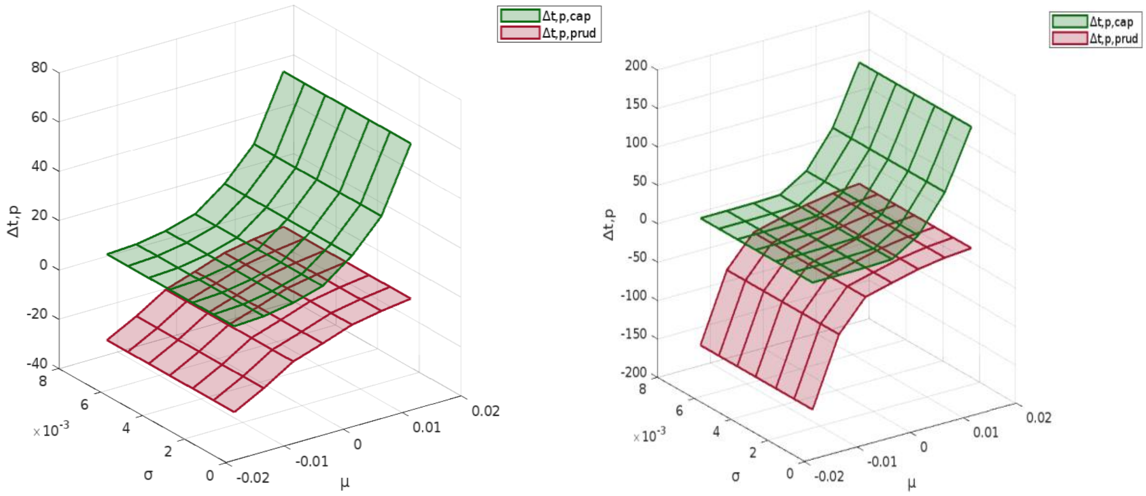
Reliability, *P*, %	**Property type**	**Growth parameter, *μ***				
		(-0/010)	(-0.005)	0	(+0.005)	(+0.010)
Adjustment parameters $\Delta_{d,p,cap,}/\ \Delta_{d,p,prud}$						
P=68.27% 1 σ	Flats	7.90/21.29	9.92/13.87	13.36/9.94	22.25/5.78	34.89/3.58
	Houses	6.59/73.03	8.70/35.73	13.33/18.82	24.4/11.57	71.57/8.25
	Land	5.64/44.39	7.83/27.08	11.59/15.41	23.86/8.27	52.94/4.24
P=95.45% 2 σ	Flats	14.37/43.05	17.27/31.62	22.62/21.59	33.41/14.6	53.71/7.41
	Houses	14.95/103.07	18.03/51.32	25.93/26.32	48.68/14.75	122.7/10.0
	Land	9.59/93.42	12.9/45.91	17.36/22.03	30.68/10.37	72.52/4.81
P=99.73% 3 σ	Flats	21.12/84.77	24.99/52.2	33.58/32.77	52.47/22.2	89.28/16.82
	Houses	19.14/123.7	22.9/59.81	33.68/30.89	68.88/15.76	153.4/11.0
	Land	18.22/116.8	22.02/53.70	30.56/24.48	43.59/11.08	104.6/4.99

Table 3. Adjustment parameters $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ level in USD/sq. m. for residential property value in UK

Reliability, *P*, %	**Property type**	**Growth parameter, *μ***				
		(-0/010)	(-0.005)	0	(+0.005)	(+0.010)
Adjustment parameters $\Delta_{d,p,cap,}/\ \Delta_{d,p,prud}$						
P=68.% 1 σ	Flats	10.15/66.85	10.60/27.74	11.75/18.97	33.41/10.6	80.90/05.19
	Houses	5.64/13.18	6.67/11.01	9.51/8.38	19.37/5.76	40.77/3.80
P=95.45% 2 σ	Flats	19.23/206.6	19.91/119.9	21.79/63.13	52.15/32.24	127.7/17.5
	Houses	11.41/25.50	16.22/20.50	20.45/15.14	39.83/12.13	83.52/7.77
P=99.73% 3 σ	Flats	36.67/343.7	38.84/175.5	45.89/92.01	109.8/53.7	269.5/36.98
	Houses	17.58/58.91	21.60/39.48	32.54/31.55	56.06/22.46	108.4/15.86

An example of general three-dimensional view of adjustment parameters $\Delta_{d,p,cap}$, $\Delta_{d,p,prud}$ dependence on market volatility σ and growth factor μ for Ukrainian and British residential property market of flats is demonstrated by fig. 5.

With tangible difference in property value expressed in most widely used metric of USD/sq.m. for British and Ukrainian markets it’s expedient to make a comparison of adjustment parameters $\Delta_{d,p,cap}$, $\Delta_{d,p,prud}$ in percentage to median property value V_0 of each property type. Such comparison shows that for confidence level 2 σ= 95.45% and with market tendency parameter μ change from -0.01 to +0.01 “cap” adjustment parameter $\Delta_{d,p,cap}$, for apartments in Ukraine is growing from 1.46% to 5.48% of initial value V_0 . For British market that adjustment parameter is changing from 0.60% to 3.52% within the same range of parameter μ.



a) Ukraine, flats *b) UK, flats*

Fig. 5. Three-dimensional dependence of the $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ parameters on the variance σ and growth coefficient μ for the upper and lower bounds of residential property in Ukraine (a) and UK (b).

Adjustment parameter $\Delta_{d,p,prud}$ for property "prudent" value estimation with parameter μ growth is decreasing as mentioned above. For the same range of parameter μ increase "prudent" adjustment for Ukrainian flats going down from 4.39% to 0.76 % of initial value of living area square meter with same confidence level at 2σ =95.45%. In case of British apartments market this change is from 5.6% to 0.48%. Similar picture can be received with slightly different figures when comparing "cap" and "prudent" adjustments housing markets in Ukraine and UK (tables 3,4).

Hence Ukrainian and British residential property markets are evolving with general tendency parameter μ close to 0 proper comparison of results for "cap" and "prudent" adjustments could be also done for that particular case (table 4).

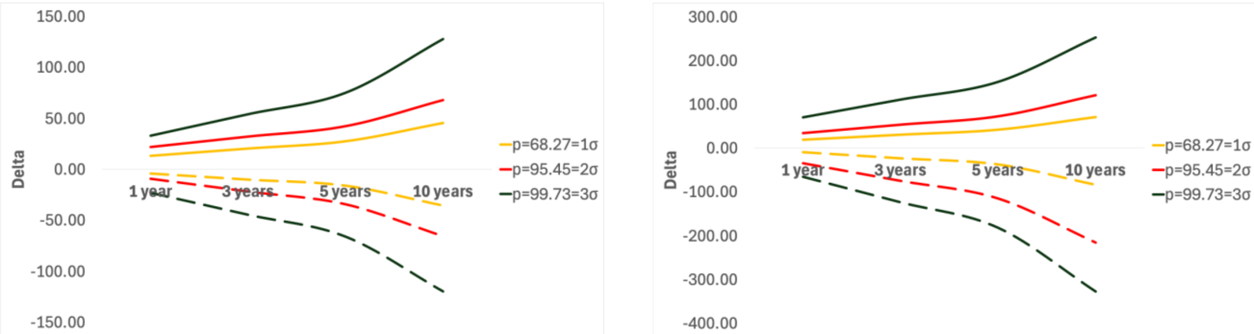
Table 4. "Cap" $\Delta_{d,p,cap}$, and "Prudent" $\Delta_{d,p,prud}$ adjustments in % of property market value V_0 for Ukraine and UK (growth parameter $\mu \simeq 0$) as of December 1, 2024.

Property Type	**"Cap" adjustment $\Delta_{d,p,cap}$,**			**"Prudent" adjustment $\Delta_{d,p,prud}$**		
	1 σ = 68.27%	**2 σ = 95.45%**	**3 σ = 99.73%**	**1 σ = 68.27%**	**2 σ = 95.45%**	**3 σ = 99.73%**
Ukrainian residential property market						
Flats	1.36	2.30	3.43	1.01	2.20	3.34
Houses	2.67	5.19	6.73	3.76	5.26	6.18
Land	2.62	3.92	6.18	3.48	4.97	5.53
British residential property market						
Flats	0.37	0.60	1.26	0.52	1.74	2.54
Houses	0.26	0.34	0.55	0.23	0.85	0.53

General comparison indicates that level of "cap" and "prudent" adjustments required in the case of the well-developed British residential real estate market is noticeably lower than for the Ukrainian one. Based on summarized results collected as an overall estimation with confidence level not less than 95 % and project duration in line with table 1 it could be recommended for the countries with developed economies to apply adjustment parameters Δd,p,cap, Δd,p,prud within the range of 1-2 % of property market value. For developing countries with greater market volatility level sufficient range of these adjustment parameters should be increased to 5-6 % of property market value (table 4).

Results discussed above are taken for time duration indicated in table 1 for comparison purposes. Following methodological steps described above and reflected in equations (4), …, (6) one can evaluate the role of the time horizon or project duration on the "cap" and "prudent" cost adjustment parameters Δd,p,cap, Δd,p,prud. Exemplified dependence of these parameters on time horizon up to 10 years is demonstrated by fig.6 for apartments in Ukraine and United Kingdom as of end 2024. Similar pattern tendency can be observed with adjustments magnitude gradual increase with time horizon distancing for other property types including houses and land plots investigated for both countries.

At the same time rate of "cap" and "prudent" adjustment parameters increase with time T is remarkably higher for bigger confidence level p needed (fig.6). Considering mentioned it should be stressed that overall recommendations formulated above should be treated as those referred to time horizon 4-5 years according with data presented in table 1 and confidence level not higher than 95 %. For shorter time horizons amount of adjustment parameters could be decreased accordingly and vice versa for longer period of time and confidence limit needed.



a) *b)*

Fig. 6. Dependence of adjustment factors $\Delta_{d,p,cap}$, $\Delta_{d,p,prud}$ on time horizon T at different confidence levels p for residential property value (flats) of 1 sq. m. in Ukraine (a) and UK (b).

Analysis conducted within the developed stochastically based analytical model of both upper or "cap" and lower or "prudent" property value estimation encompasses following key variables: property market value general trend μ, its volatility degree σ, confidence level required p and time horizon T needed. Extensive calculations provided gave a possibility to create multi-factor regression models for adjustment parameters $\Delta_{d,p,cap}$, $\Delta_{d,p,prud}$ estimation that embrace role of all these most important factors for all types of residential property at Ukrainian and British markets analised.

Resulted regression type equations received are as follows:

- for upper or "cap" bound adjustment parameter

$$\Delta_{t,p,cap} = -4319.664 * \sigma^2 + 18342.198 * \mu + 1.04 * t + 2.18 * p - 175.32 \quad (12a)$$

-for lower or "prudent" value adjustment parameter

$$\Delta_{t,p,prud} = 2844.88 * \sigma^2 - 153.85 * \mu - 0.96 * t - 1.79 * p - 204.63 \quad (12b)$$

Statistical outputs of regression analysis provided when deriving equations (12a), (12b) are summarized in table 5. Both models have been validated and showed statistically significant relationships between the adjustment parameters and their predictors being confirmed by high F-criteria values and low significance levels (table 5).

Table 5. Summary output for multi-factor dependence models 12a, 12b of adjustment parameters $\Delta_{d,p,cap}$, $\Delta_{d,p,prud}$

Dependence model	*Factors*	*Coefficients*	*Standard Error*	*t Stat*	*P-value*
$\Delta_{t,p,cap}$: Signif. F = 3.0062E-58 Multiple R = 0.916667	Const.	-255.30869	39.4822891	-6.4664105	2.1272E-10
	σ^2	-4319.664	1108.90244	8.1063335	3.0912E-15
	μ	18342.198	592.301856	15.7910504	5.1692E-47
	T	1.03833362	0.14464254	7.17861874	2.1574E-12
	p	2.17672692	0.41671887	5.22349021	2.4489E-07
$\Delta_{t,p,prud}$: Signif. F = 8.451E-81 Multiple R = 0.894395	Const.	-255.30869	39.4822891	-6.4664105	2.1272E-10
	σ^2	-4319.664	1108.90244	8.1063335	3.0912E-15
	μ	18342.198	592.301856	15.7910504	5.1692E-47
	t	1.03833362	0.14464254	7.17861874	2.1574E-12
	p	2.17672692	0.41671887	5.22349021	2.4489E-07

The statistically significant relationships observed between the correction parameters and key variables such as market volatility *(σ^2),* growth rate *(μ),* probability level *(p),* and time *(t)* suggest that these models can be effectively used to predict fluctuations in residential property valuations. The strong explanatory power, indicated by high F-statistics and low significance F values, confirms that these factors play a crucial role in determining property price adjustments.

Additionally, the presence of both positive and negative coefficients across different variables reflects the dynamic nature of the property market, where certain factors drive value increases while others contribute to market corrections. The coefficients reflecting market volatility *(σ)* and growth rate *(μ)* have opposite signs in these two models, highlighting the different influences these variables have on the respective correction factors. Time *(t)* and probability level *(p)* also exhibit differing impacts on the correction factors in the two models, showcasing the nuanced dynamics of the residential property valuations mentioned above.

It should be underlined that summarizing equations (12a), (12b) embrace different types of residential property and at markets with quite distinguishing economies. The role of two critically influential factors in adjustment parameters needed to determine property “prudent” or “cap” value is reflected by in the magnitude of the volatility (σ) and general market trend (μ) parameters. The remaining important parameters, including the time horizon (T) and the confidence level (p) are subject to the objectives of the analysis. In this sense, equations (12a), (12b) can be considered as generalized for a wider area of application for estimation of property upper or “cap” value and lower or “prudent” value adjustment parameters.

5. Concluding remarks

Summarized stated above, it should be mentioned that manuscript presented extends results presented in 2 previous papers [Yakubovsky, et al, 2022: 19, Yakubovsky, et al, 2024: 20] devoted to development of new analytical approach and model for estimation of upper or “cap” and lower or “prudent” property value stochastically evolving over time. This third publication on the topic enlarges previous ones encompassing also analysis and reflection of time horizon role in necessary adjustment parameters levels. Moreover, based on detailed analyses of Ukrainian and British residential property market provided generalized multi-factor regression equations proposed that reflect main market particulars, i.e. property general market trend, its volatility level, time horizon needed and confidence level required. With the possibility of wider application, it emphasizes the advantages of the developed approach based on a solid theoretical basis and could be considered as an example of one more practical application of Samuelson’s Rational Theory of Warrant Pricing.

From applied side it also should be stressed that developed model satisfies main recent requirements set by international financial institutions for property prudent valuation for collateral purposes, hence:

- it gives an adjustment parameters magnitudes that does not allow for current property value level be below market value during project term with given level of confidence when determining property prudent value;
- property market value is used as reference and basis for estimation of its prudent value;
- it reflects role of such important parameters as project duration and level of confidence needed;
- it’s based on analysis and generalization of existing market evolution increasing reliability of assessment provided and generalizations made.

Generalized multi-factor regression type equations received based on conducted detailed analysis open an opportunity for their broader implementation in respect to other markets and property types for estimation of stochastically evolving over time their value upper and lower extremes.

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