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Generalized Stochastic Model of Long-Term "Cap" and "Prudent" Value Estimation

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Abstract: The objective of this study is to analyze existing approaches to the effect of property value fluctuations over time and develop an analytically based approach to consider its significant role. Samuelson's Rational Theory of Warrant Pricing is used as the theoretical foundation for estimating upper ("cap") and lower ("prudent") property values. Analytical decisions received for the estimation of correction parameters necessary to arrive with a certain level of reliability to both the upper and lower bound of property value stochastically evolving over time. The practical application of the developed generalized methodology is demonstrated using an available residential property database. A generalized analytical approach for estimating property value lower and upper bounds is proposed and tested, combining initial market value, its general trend over time, volatility level, and required estimation reliability. The study investigates the role of these main parameters in determining the property value bounds. This research presents and describes a novel stochastic, analytically based methodology for estimating the evolving lower and upper bounds of property values over time. The results significantly contribute to existing methodologies for estimating both "prudent" and "cap" property value levels. The model development, testing, and results analysis are based on a residential property market dataset.

Keywords: Stochastic model, value volatility, prudent value, cap value, long-term value, Markov processes, correction parameter, model testing, value estimation.

1. Introduction

Property valuation is commonly used across the financial system playing pivotal role of such processes as company listings, mergers and acquisition, funding and investments, financial reporting, auditing, secured lending, taxation, insurance, litigation, insolvency and many others. This confirms importance of robust and reliable valuation approaches, methods and practices for the global financial system stable operation as underlined by the International Valuation Standards Council [1].

Following the requirements of all internationally recognized valuation standards, all key basis of value which stipulate fundamental premises on which the reported values of property are based such as market value, equitable value, investment value, etc. should be determined at a specific valuation date. As an example, a definition of most widely used market value states that “market value is the estimated amount for which an asset or liability should exchange on the valuation date between willing seller and willing buyer…” It means that property under consideration appropriate value should be determined and hence should corresponds to this fixed period of time accordingly [2].

But with market evolution under the influence of broad spectrum of acting exogenous and endogenous factors, this value will vary over time and for any other time point it might be either higher or lower of the estimated level for this fixed point of time, being only occasionally at the same level. Such fluctuations of assets values are unavoidable in market economies with stochastic processes of their evolution, creating a problem of estimation a proper level of values for applications where time effect is essential. Actually, requirements to estimate property value at fixed point of time could be considered as an indirect recognition that time matters in valuation processes.

These phenomena of time effect are well understood and recognized as being important. Most recent significant impetus in this sense was given by the last Global Financial Crisis 2007-2009. General importance of the reliable valuation was strongly confirmed also by generalized data from US, Europe and Japan, which identify that a half of the fall in the value of banks during Global Financial Crisis and the most recent COVID-19 effect was caused by the level of falls in Real Estate Investment Trust prices [3].

In response to the deficiencies in financial regulations revealed by this global crisis, the third installment of the Basel Accords (Basel III) in particular stated that “…valuation must be appraised … using prudently conservative valuation criteria…” [4]. Within the EU, the Basel III guidelines have been implemented through the EU Regulation 575/2013 on Prudential Requirements for Credit Institution and Investment Companies [5] and the EBA Regulatory Technical Standards 2012&2020 [6, 7]. As a result, Basel III Accord and European Banking Authority (EBA) emphasized that financial institutions should not be allowed to apply solely market value or fair value concept in assets valuation practice.

Implementation of these prudential requirements is recognized as an important measure to ensure efficiency of banking and other financial services functioning and are meant to ensure the financial stability of the operators on those markets with high level of investors interests protection. As a result, Basel III Accord and European Banking Authority (EBA) emphasize that financial institutions should not be allowed to apply solely “market value” or “fair value” concept in assets valuation practice.

2. Object and subject of research

To secure stability and resilience of financial system regulatory requirements mentioned above are addressed in first hand to cautious or “prudent” estimation of property value with focus to lower bound value in addition to standardized “market value” approach. In addition to lower bound or “prudent” value an upper or “cap” bound value estimation for several important practical applications should be also considered as essential. Any investor for example before final investment decision to be taken might be interested in getting maximum level of resources needed in course of project duration. Same is in case of taxation application, mergers and acquisitions, litigation purposes as well. Hence, for evolving over time property value not only lower bound is of interest and importance but also its upper bound that could be addressed as “cap” value. And most preferable is general methodology that encompass possibility to assess on the same platform both upper and lower value change boundaries for the given period of time.

At the same time property value is evolving over time within its lower and upper bound stochastically. This is quite natural because of the fact that this evolution is taken place under the influence of broad spectrum of endogenous and exogenous factors stochastically changing over time in open markets. That leads to thoughts on expediency of probabilistic approach foundation for estimation of value evolution boundaries to be applied.

All these particulars clearly indicate timeliness and expediency to consider time effect role in processes of property value estimation with defining not only mean, median or most probable value parameters but also some additional metrics that can characterize level of expected value volatility. And that is especially important for many practical applications where possible changes in value during project life are valid playing essential role over time. With respect to the stochastic nature of value evolution over time to be reliable and trustable such methodology of assessing this additional metrics should be based on the concepts of probabilistic analysis.

3. Target of research

Coming from this, the paper’s main focus is oriented to review current status of time effect consideration in the assets long-term value estimation and propose a new generalized analytical approach of estimation both low or “prudent” value bound and its upper or “cap” counterpart with testing of the results achieved using market evolution data set available.

4. Literature analysis

Gradual recognition of not only usefulness but a real need for involvement of probabilistic analysis to the value estimation passed a long way in a history of valuation profession foundation [8]. This statement could be treated as a historical anticipation of Giuseppe Medici’s probably first time clearly originated concept of “most probable market price” which appeared in 1953 or seven centuries after to replace traditionally used at this time notion of the “highest price” [9].

Regardless of the fact which valuation approach and particular method or even combination of methods has been used value estimation is always a forecast by definition. And as any forecast valuation process is constantly facing uncertainty which is coursed by the influence of broad spectrum of influencing factors evolving stochastically. It leads to evident conclusion that assets value estimated on the basis of market prices in particular within comparative and cost approaches directly and within income approach indirectly via cash flows are random variables and should be treated as such.

This concept of probabilistic nature of value estimation was strongly advocated and further enhanced by Richard U. Ratcliff [10], James Graaskamp [11] and William Kinnard [12]. More recently this concept was enlarged by Max Kummerov [13] and Gale L. Pooley [14]. Henry Babcock in his widely recognized textbook of 1968 also voted in favor of “most probable buy-sell price” as being central for the notion of market value [15].

In addition to the concept of “most probable price” Ratcliff proposed simultaneously to express value in probabilistic terms and more specifically as 90 % confidence limit range [10]. And finally, notion of market value was fixed in the latest editions of the International Valuation Standards as “… the most probable price reasonably obtainable in the market on the valuation date …” [2].

Responding to practical needs mentioned above several approaches to take into account time effect role in property value estimation have been developed. With respect to the cyclical mode of value change over time two main concepts have been identified [16]. The first one is “through-the-cycle” type which averages or “flattens” the value pattern fluctuations through the time period, aiming to identify a fair economic or equilibrium value. The main application of such model covers mostly investment type of financial analyses and general decisions with time effect concern (fig.1).

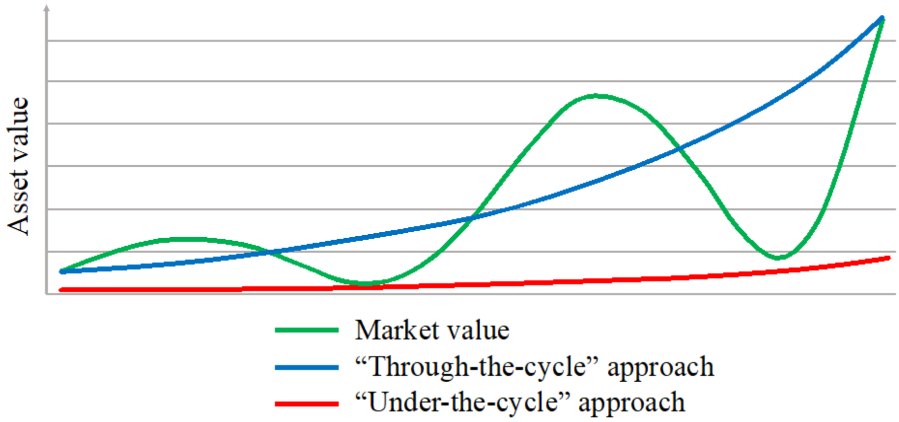

Figure 1. General scheme of “through-the-cycle” and “under-the-cycle” approaches.
Source(s): Author's own creation

The second one is “under-the-cycle” concept which intends to find lower “skirting” line of market value fluctuations over a given period of time (fig.1). This second type of concepts reflects more closely the Basel III requirements to implement prudent assessment of assets value with respect to a long-term effect. This “prudent” long-term value should not exceed the market value at any time under consideration in accordance with these recommendations.

In technical literature devoted to property valuation with respect to the time effect consideration authors failed to find out any methodology to define value upper bound to arrive to its value “cap” metrics.

Most common “through-the cycle” type of approaches for the time effect consideration are those described by Nordlund [17] and Cardozo [18] being named as “reference value model” and “adjusted market value-AMV”, respectively. Both of them are grounded on the previous statistical data through identification of long-term trends based on past data available, assuming that the future would look like the past.

More recent AMV- model is grounded on comparing asset’s current market value as reflected in an appropriate capital value index to a long-term trend line. The regression-generated, long-term trend line is drawn dynamically rather than with historical hindsight through an inflation-adjusted capital value index [18].

More sophisticated “through-the-cycle” models include some consideration of the trend extension to the future with reference to the current value, with forecast of its change in coming years. This forecast is mainly based on conventional discounted cash flow (DCF) models [19, 20, 21].

Among “under-the-cycle” approaches, the longest history of development belongs to the Mortgage Lending Value (MLV) approach developed primarily in Germany as far back as the beginning of this century [22]. This concept development and implementation was mainly driven by the Association of German Mortgage Banks (German Pfandbrief Banks-VDP) in a close cooperation with the Federal Financial Supervisory Office. Being most well established, this approach came into force in 2005 through the adoption of the German Pfandbrief Act, which regulates the determination of the mortgage lending value [22].

From that time this approach plays a central role in the property valuation for lending purposes in Germany. Several European countries, which to some extent follow the Germany experience in this respect, are Austria, Czechia, Hungary, Luxemburg, Poland, Slovenia and Spain [23].

The definition of the Mortgage Lending Value is set out in the EU Capital Requirements Regulation (CRR) No.575/2013 as “the value of immovable property as determined by a prudent assessment of the future marketability of the property taking into account long-term sustainable aspects of the property, the normal and local market conditions, the current use and alternative appropriate uses of the property” [5].

In its essence MLV model is principle based. Being the only long-term value concept that has been developed and adopted for the secured lending purposes, its principal base led to the fact that application of the concept in different jurisdictions vary [24]. Diversity of this model application in different countries creates certain complications of comparative nature and calls for unified approach needs.

When deriving the property MLV, current transaction databases available should be accompanied by a broader analysis from the past as well as a reasonable forecast analysis to be added to arrive to the “prudent” sustainable value. Hence it is based on a combined consideration of available historic empirical data, current market values and future assessments, using analyses and forecasts that ignore short-term price volatility to arrive to a realistic lowest possible level of market value, which is expected in a future time horizon under consideration.

More structured approach of prudent value estimation is given by the EBA Regulatory Technical Standard for prudent valuation that constitutes the EU Prudent Valuation Framework [7]. In general sense the CRR requires from institutions a prudent valuation of assets measured at fair value and the deduction of the resulting Additional Valuation Adjustments (AVAs) from the Common Equity Tier One (CET1) capital [5].

The EBA RTS [7] allows application of two approaches to prudent valuation. The simplified approach, applicable for financial institutions with total absolute fair-valued assets and liabilities below EUR 15 bln prescribes total AVA equal to 0.1% of the total fair value. The core approach, compulsory for institutions above the EUR 15 bln threshold, prescribes the calculation of 9 AVAs, referring to most influential sources of valuation uncertainty to achieve the prudent value with 90% level of confidence.

Where possible calculation of respective AVAs should be based on a market data and specified level of certainty. In case of lack of market data expert-based approach is specified with the same 90% level of certainty. To consider correlation between different uncertainties considered aggregation factor is provided when arriving to AVAs total level. Initially fixed at the level of 50% this factor was raised afterwards to 66% [7].

It should be also mentioned that all “under- the-cycle” type of approaches in general sense could be also applied as the scheme for estimation of value upper bound or “cap” value for the period of time under consideration. It could be done using so called mirror scheme in relation to “prudent” value estimation basics with some additional adjustments necessary. To the best authors knowledge, no indication and methodology to establish on the same principles stochastically evolving property value upper bound or “cap” value are available in dedicated technical literature.

With the certain individual particulars in place one of the main commonalities of all long-term “prudent” value estimation methodologies are the schematization of value evolution pattern over time as demonstrated by fig.1 when assets market or fair value should be corrected to reach its long-term “prudent” value by deduction of certain adjustments. Such adjustments are not analytically and statistically grounded being mainly based on expert’s opinion rather than analysis of real property value fluctuations over time. It also indicates necessity to develop more reliable and trustable analytically grounded models for assessing both upper and lower extremes of market value stochastically evolving over time. The need of developments in this area was strongly underlined by the international group of experts representing most recognized international valuation professional bodies [24].

With regard to the nature and character of value evolution over time it leads to necessity of calling in the theory of stochastic processes for solving this practically needed task in developing generalized stochastically grounded approach for estimation of both lower bound or “prudent” and upper bound or “cap” assets value. This thesis constitutes the main objective and simultaneously main content of the publication presented.

5. Research methods

In rather broad spectrum of stochastic processes family most appropriate for the main purposes stated above are processes of Markov type. The main reason for that is the fact that mathematical background of Markov processes has been most fully developed in comparison with other types of stochastic processes [25]. This advantage creates a good prospects of Markov processes theory application for efficient solving a wide range of practical problems in different spheres.

The history of Markov processes application in economic application can be traced back as far as very beginning of previous century when French mathematician Louis Bachelier used a Wiener process to model price changes on the Paris Bourse, a stock exchange [26]. This publication is now well considered as pioneering one in the field of financial mathematics.

Some other important historical track of Markov processes application in econometrics includes analysis of income distribution [27], the size variability of firms, asset prices volatility and market crashes [28], GDP evolution, etc. [29]. More recent examples are switching multifractal model developed by Laurent E. Calvet and Adlai J. Fisher [30], which uses an arbitrarily large Markov chain to drive the level of volatility of asset return and stock volatility analysis in relation to equity premium provided by Michael Brennan and Xiab Yihong [31].

One of the most important advantage of Markov processes is that general theoretical decision estimating time parameters when such stochastic processes may reach low, upper or any other boundary is given [32]. This opportunity opens the principal way for solving a problem under discussion for finding out lower or "prudent" and upper or "cap" value evolving stochastically.

At the same time analytical decisions on Markov process approach to the boundaries are usually focused on estimation of probability distribution function of the first-time approach to certain boundary or first two moments of that parameter, i.e. mean time and dispersion [33]. It creates a certain difficulty because we need to solve opposite task being interested to find out not a probabilistic parameters of first-time boundary approach but rather magnitude of the boundary for Markov process evolving over time horizon subject of analysis.

In general, solving of this task involves two subsequent stages. The first one is related to proper analytical description of the asset value stochastic behavior. That gives a possibility to find out at the next stage estimation of that process upper and lower boundary magnitude for the given period of time with given level of probability.

6. Research results

Among analytical decisions on description of stochastic processes in economics probably most well-known and recognized is that given by the Rational Theory of Warrant Pricing developed by Paul Samuelson [34]. Introducing the notion of nonnegative "economic" Brownian motion he substantially modernized and improved description of such stochastic processes. That was followed by development of complete theory of warrant pricing.

"Diffusion" type model of the market developed has found widespread use in stochastic financial mathematics during last decades. This model is associated with the well-known results of Black and Scholes [35], Merton [36], Bensoussan [37], Karatzas and Shreve [38] in different important branches of post-neoclassical economics.

Following the notation given by Shiryaev, *et al* [43] we can describe stochastic evolution of the asset value V_t by the following equation of diffusion type:

$$V_t = V_0\, exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right\}, t \geq 0. \tag{1}$$

Here, $V_0 > 0$ is asset value at $t = 0$ point of time being non-random and known, $\mu \epsilon R$ is growth factor, $\sigma > 0$ is process volatility, and W_t is a Wiener process with continuous paths.

According to equation (1), the asset value evolution V_t depends primarily on three key parameters: growth factor μ, volatility coefficient $\sigma > 0$, and "randomness" of Weiner process determined by function W_t. Actually, by its nature equation (1) gives a description of asset value stochastic evolution over time with these main parameters to be estimated based on analyses of value process evolution data.

In the present case of the process lower and upper bounds estimation, a rationale for the model (1) is as follows. Rewriting equation (1) in terms of log-values we obtain:

$$V_t = V_0 + at + \sigma W_t, t \geq 0 \tag{2}$$

Where:

$$a = \mu - \frac{\sigma^2}{2}, t \geq 0 \tag{3}$$

With given current value V_0, probability p and time horizon $T > 0$, our goal is to find $\Delta_{t,p}$ such that:

- for lower bound – $P\{min\, V_t \geq (V_0 - \Delta_{t,p})\} = p, 0 \geq t \geq T;$ (4)

- for upper bound – $P\{max\, V_t \leq (V_0 + \Delta_{t,p})\} = p, 0 \geq t \geq T;$ (5)

Taking into account an expression for the running supremum of a scaled Brownian motion with linear drift [40], for any $\Delta_{t,p,cap}$ in case of upper bound with the use of equations (2), (3) and (5) we have:

$$P\{\max V_t \leq (V_0 + \Delta_{t,p,cap})\} = P\{\max(at + \sigma W_t) \leq \ln(V_0 + \Delta_{t,p,cap}) - \ln V_0\} = \\ - \exp\left(\frac{2ay}{\sigma^2}\right)\left(1 - \Phi\left(\frac{y+aT}{\sigma\sqrt{T}}\right)\right) + \Phi\left(\frac{y-aT}{\sigma\sqrt{T}}\right), \tag{6}$$

where $y = \ln(V_0 + \Delta_{t,p,cap}) - \ln V_0$ and Φ stands for standard normal cumulative distribution function.

If for a given $p \subset (0,1)$, $y_p \geq 0$ is a real number such that:

$$- \exp\left(\frac{2ay_p}{\sigma^2}\right)\left(1 - \Phi\left(\frac{y_p+aT}{\sigma\sqrt{T}}\right)\right) + \Phi\left(\frac{y_p-aT}{\sigma\sqrt{T}}\right) = p \tag{7}$$

then $\Delta_{t,p,cap} = V_0(e^{y_p} - 1)$ satisfies the equality $P\{\max V_t \leq (V_0 + \Delta_{t,p,cap})\} = p$, because: $\ln(V_0 + \Delta_{t,p,cap}) - \ln V_0 = \ln((V_0 + V_0(e^{y_p} - 1)) = \ln(V_0 e^{y_p}) - \ln V_0 = y_p$.

Thus $\Delta_{t,p,cap} = V_0(e^{y_p} - 1)$ is a required solution of:

$$P\{\max V_t \leq (V_0 + \Delta_{t,p,cap})\} = p. \tag{8}$$

Using similar approach, we can arrive to the following estimation of the correction parameter $\Delta_{t,p,prud}$ for the value evolution lower bound estimation which has the following form $\Delta_{t,p,prud} = V_0(1 - exp^{x_p})$.

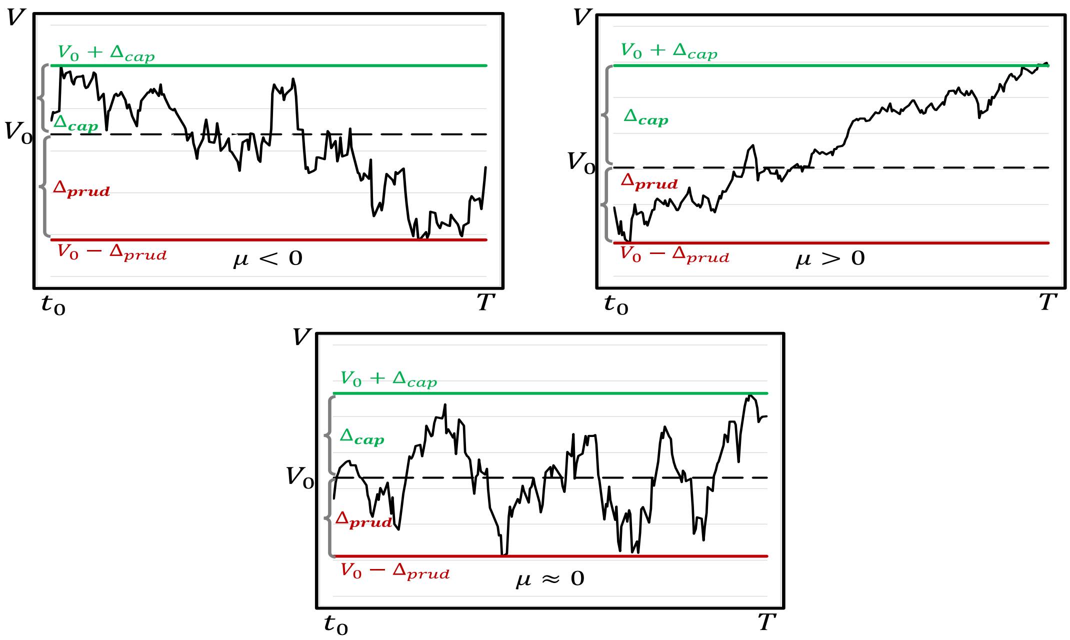

Figure 2. Schematic presentation of long-term “caps” and “prudent” value estimation for growing (μ>0), stagnating (μ~0) and falling (μ<0) markets.

Source(s): Author's own creation

Finally, for the asset value stochastic process evolution lower or “prudent” $V_{t,p,prud}$ and upper or “cap” $V_{t,p,cap}$ value magnitudes can be estimated as:

$$V_{t,p,prud} = V_0 + \Delta_{t,p,prud} = V_0 + V_0(1 - exp^{x_p}), \quad (9.1)$$

$$V_{t,p,cap} = V_0 - \Delta_{t,p,cap} = V_0 - V_0(exp^{y_p} - 1), \quad (9.2)$$

where V_0 is the asset value at the process analysis starting point when *t=0*, parameters x_p and y_p stand for:

$$x_p = \ln\left(V_0 - \Delta_{t,p,prud}\right) - \ln V_0, \quad (10.1)$$

$$y_p = \ln\left(V_0 + \Delta_{t,p,cap}\right) - \ln V_0, \quad (10.2)$$

Equation (9) gives a final version of asset lower $V_{t,p,prud}$ and upper $V_{t,p,cap}$ bound value estimation with correction parameters $\Delta_{t,p,prud}$ and $\Delta_{t,p,cap}$ estimation for the asset with initial market value V_0, given level of probability p and assets market value stochastic evolution parameters σ, μ encompassed by parameters x_p and y_p. Detailed practical application of the theoretical model presented is given below with graphical presentation of the scheme demonstrated by the Fig.2.

To provide model developed testing and its parameters assessment an initial statistical data base of asset market evolution is necessary. For this purpose, Ukrainian residential property market initial data set which is monitored and fed on a constant basis under the authors methodological and operational guidance is used.

The creation of this primary database is carried out by monitoring and accumulating information flows from the existing real estate market and their subsequent in-depth statistical processing. The use of modern methods of database management (PostgreSQL), geo-information systems (QGIS) and scripting library (Python) allows to arrange data base formation and continuous updating initial data set most efficiently.

Appropriate automated calculation tools, as well as application program packages, were utilized to reach comprehensive results in a systematic manner, enabling the identification of modern real estate market trends and the forecasting of priority directions for future development. A noteworthy aspect of this analysis is its fully probabilistic nature, which is essential for reliable assessment of the model developed parameters assessment. Methodological basis and main elements of the residential property data base which covers all regions of the Ukrainian property market as well as its evolution particulars are described in dedicated publication [41].

Time range covered by this primary data base of residential apartments used for the assessment of the parameters was 54 months from July 2019 till December 2023. This period includes also two COVID-19 pandemic outbreaks fixed in Ukraine in spring 2020 and autumn 2021 as well as continuing war started February 2022. Overall monthly size of the market available for Ukraine totally and tailored in this data base fluctuated between 175,000 to 210,000 sales propositions all over the country.

A statistical analysis of residential property market representative samples was performed first of all in relation to the widely used main financial indicator which is the cost of 1 sq. m. of the apartment’s area. Multiple application of powerful Pearson’s statistical agreement criteria as for overall country data set as well as for separate regions and cities within country, there clusters as well as different time intervals demonstrated clearly the closest compliance of the general statistical sample distribution of this main financial indicator with log-normal distribution law (Fig.3). This important statistically based conclusion is compliant with one of the basic assumptions of Samuelson’s Rational Theory of Warrant Pricing [34]. Based on this, all further processing of primary information database was grounded on determining the parameters of the log-normal distribution adopted as theoretical law for the whole general population of information databases of the distribution of apartments square meter area value.

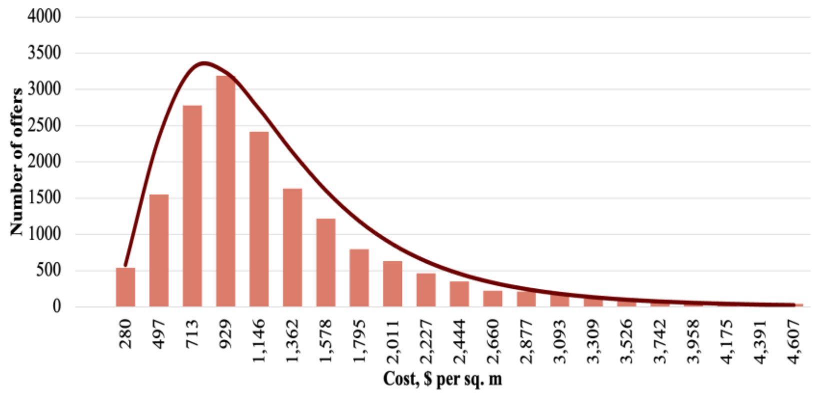

Figure 3. Density of distribution of apartments 1 sq. m. cost in the secondary market of Ukraine as of December 2023.

Source(s): Author's own creation

Evolution of median value V_{av} and the coefficient of variance σ_v of the square meter apartment’s area for the overall country market demonstrate evolution of these parameters over time (Fig. 4). General tendency of the median value change within time frame indicated is quite evident being oriented to growth, whereas the volatility of this parameter measured through the coefficient of variance is more stable in time last years.

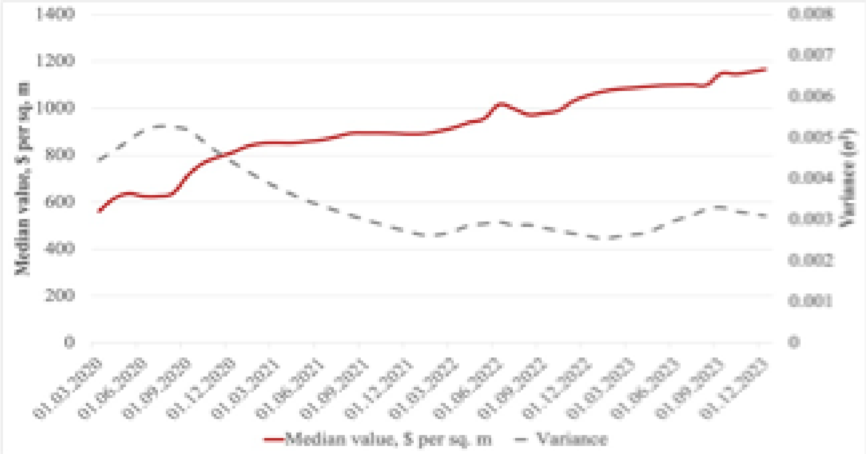

Figure 4. Dynamics of median and variance levels of housing square meter cost in Ukraine in 2020-2023.

Source(s): Author's own creation

In our practical case for calculation purposes we have: time interval τ=1 month, number of intervals n=54, and total duration T=54 months. Based on data set available, we estimate parameters μ and σ using log-returns as follows.

$$q_k := \ln\left(\frac{U_{k\tau}}{U_{(k-1)\tau}}\right), k = 1,\ldots,n. \tag{11}$$

The log-returns q_k follow a distribution expressed as:

$$\frac{q_k}{\sqrt{\tau}} = a\sqrt{\tau} + \sigma\gamma_k, k = 1,\ldots,n, \tag{12}$$

where $\gamma_k = \tau^{-\frac{1}{2}}\left(B_{k\tau} - B_{(k-1)\tau}\right), k \geq 1$, are independent standard normal variables.
For the model of observations (12), the maximum likelihood estimator of a is given by:

$$\hat{a} = \frac{1}{n\tau}\sum_{i=1}^{n} q_i = \frac{1}{n\tau}\ln\left(\frac{U_{T_0}}{U_0}\right), \tag{13}$$

and the unbiased estimator of σ^2 is computed as:

$$\hat{\sigma}^2 = \frac{1}{(n-1)\tau}\sum_{i=1}^{n}\left(q_i - \tau\hat{a}\right)^2. \tag{14}$$

According to relation (3), μ is determined as:

$$\hat{\mu} = \hat{a} + \frac{\hat{\sigma}^2}{2}. \tag{15}$$

In the ensuing steps, we substitute the maximum likelihood estimator $\hat{a}$ and $\hat{\sigma} = \sqrt{\hat{\sigma}^2}$ for a, σ, respectively, within the given expressions. It is imperative to underscore that the methodology for deriving the estimator x_p is intricately detailed in [42] and is based on obtaining x_p using the following equation (16), where for any $T > 0$, $a \in \mathbb{R}$, $\sigma > 0$ and $x_p \leq 0$:

$$F(x_p; a; \sigma) := P\left(\min_{0\leq t\leq T}(at + \sigma W_t) \geq x_p\right) = -\exp\left(\frac{2ax_p}{\sigma^2}\right)\Phi\left(\frac{x_p + aT}{\sigma\sqrt{T}}\right) + \Phi\left(\frac{-x_p + aT}{\sigma\sqrt{T}}\right) = p \tag{16}$$

After determining x_p, we can compute the parameter $\Delta_{t,p,prud}$ as a time factor based on equation (9), where

$$\Delta_{t,p,prud} = V_0(1 - exp^{x_p}). \tag{17}$$

Concurrently, the computation of y_p is systematically conducted through the analytical solution of equation (7). Here we subsequently apply formula (9) to ascertain the parameter $\Delta_{t,p,cap}$ as:

$$\Delta_{t,p,cap} = V_0(exp^{y_p} - 1). \tag{18}$$

This nuanced and systematic approach distinguishes the methodologies employed for each estimator. Subsequently, the culmination of this methodology yields the final correction parameters for both the upper bound ($\Delta_{t,p,cap}$) and the lower bound ($\Delta_{t,p,prud}$) of apartments square meter area value.

By leveraging statistical calculations based on a dataset available at $t = 0$ corresponding to December 1, 2023, the specific value V_0 at that moment is determined to be 1125 USD per sq. m. Parameter estimations for μ, σ and a were conducted over a 54-month period from July 2019 to December 2023, employing formulas (13-15) with the following results:

$$\hat{\mu} = 0{,}012808638, \hat{\sigma} = 0{,}055188773, \hat{a} = 0{,}011285738.$$

The calculation of the time factors $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ involved the selection of three distinct probability values: : $p_1 = 0.6827, p_2 = 0.9545, p_3 = 0.9973$. These probabilities adhere to the n-sigma rule, where, for a distribution $\gamma \sim N(m, s^2)$, the relation $P(|\gamma - m| \leq ns) = p_n$ holds true, with $n = 1, 2, 3$. Subsequently, for each probability p_n, the value of $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ were computed using equations (17) and (18) respectively.

Fig. 5 gives an overall dependence of time factors $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ on the declared growth parameter μ. With the same level of volatility $\hat{\sigma}$, the parameter μ varies between (-0.010) and the previously estimated from historical data set value of (+0.0128), and we set $a = \mu - \frac{\hat{\sigma}^2}{2}$. Here, positive μ corresponds to a growing market, $\mu = 0$ to a stagnated market, whereas $\mu < 0$ to a decreasing or “falling” market.

As expected, the general tendency indicates that for larger levels of growth factor μ, the correction parameter $\Delta_{t,p,cap}$ is bigger to reflect influence of general trend role in addition to the level of volatility influence at fixed period of time. For lower bound correction parameter $\Delta_{t,p,prud}$ this tendency is opposite hence negative trend requires more impressive correction for lower bound of stochastically evolving process (Fig.2).

At the same time for stronger requirements as for results reliability expressed by the probability level p, the stochasticity correction parameter also demonstrates respective growth (Fig.5).

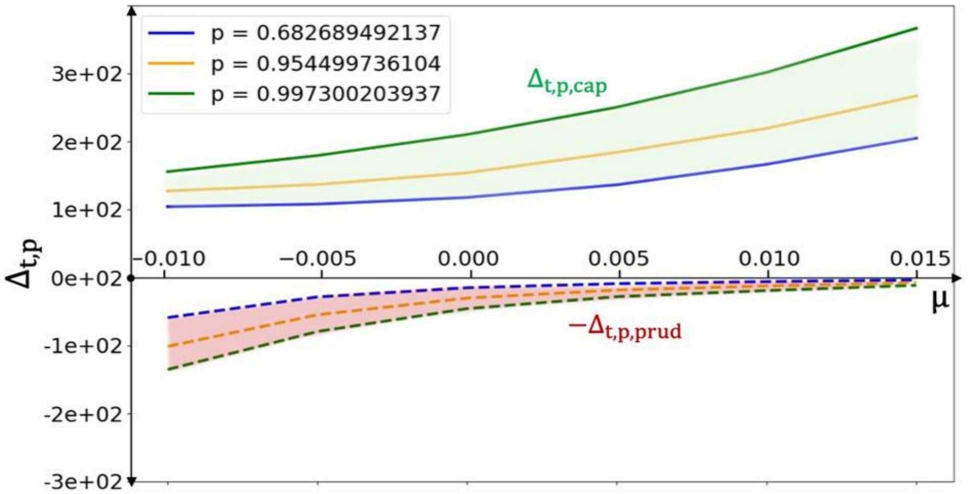

Figure 5. Dependence of time correction factors $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ on growth factor μ for different levels of probability p residential property value of 1 sq. m

Source(s): Author's own creation

Several numerical results received for different levels of expected market growth factor μ and accepted probability level p, which confirm the general tendencies mentioned above, are given in Table 1 below. As clearly seen from results presented stochasticity correction parameter magnitude for upper bound is remarkably higher in comparison with its level for the lower bound when other parameters being equal. Most evident it's seen for stagnating markets when growth trend μ is close to zero. It corresponds to reality being in line with the character of housing general probability distribution function demonstrated by Fig.3 when upper distribution "tail" is pronounced.

Strong dependence of correction parameter Δ on expected growth rate μ is also clearly seen with opposite tendencies for upper bound and lower bound. For growing markets long-term lower bound correction parameter of market value is not impressive when for suffering economies this correction should be high enough to cover their "falling" markets trend. In opposite higher bound correction parameter is much more significant for growing markets being less impressive for the case of strongly suffering markets.

Given opposite tendency proper comparison of time effect correction necessary could be done comparing the results for stagnating markets with growth rate close to $\mu = 0$. For such case long-term correction to get asset prudent value $\Delta_{t,p,prud}$ deviate within the range from 14.99 USD/sq. m. to 45.70 USD/sq. m. depending on reliability level required. For the same reliability levels upper bound correction parameter $\Delta_{t,p,cap}$ spans from 117.87 USD/sq.m. to 210.87 USD/sq.m. (Table 1). To compare with median market value of Ukrainian residential apartments data base demonstrated at Fig.3 this range in percentages for prudent value correction factor constitutes from 1.4 % to 4.2 % and for cap value correction parameter from 11.1 % to 19.8 % accordingly for different reliability levels required for the year 2023 end results.

Table 1. Correction factors $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$* level in USD/sq. m. for residential apartments expected market growth level μ and given level of reliability p

Reliability level, p, %	Growth factor, μ					
	(-0.010)	(-0.005)	0	(+0.005)	(+0.010)	(+0.015)
p=68.27=1σ	$\frac{104.36}{58.89}$	$\frac{108.08}{28.60}$	$\frac{117.87}{14.99}$	$\frac{136.56}{8.94}$	$\frac{166.63}{5.97}$	$\frac{205.00}{3.58}$
p=95.45=2σ	$\frac{127.32}{101.29}$	$\frac{136.88}{54.85}$	$\frac{153.97}{30.22}$	$\frac{184.11}{18.31}$	$\frac{219.32}{12.35}$	$\frac{267.03}{7.41}$

Continued Table 1

p=99.73=3σ	$\frac{155.67}{135.33}$	$\frac{179.33}{79.41}$	$\frac{210.25}{45.7}$	$\frac{250.49}{28.15}$	$\frac{301.62}{19.11}$	$\frac{366.57}{11.47}$

*In the table, the numerator represents the value of $\Delta_{t,p,cap}$, while the denominator represents the value of $\Delta_{t,p,prud}$.

Source(s): Author's own creation

Three-dimensional presentation of $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ correction parameters dependence from tendency growth rate μ and volatility level σ for the fixed reliability level p=95.45=2σ is given by fig.6 where general tendencies of their magnitudes changes described above are clearly demonstrated.

Most adequate comparison of developed analytical approach results with existing recommendations available could be performed in relation to the regulatory requirements established by the CRR/EBA model mentioned above. According with requirements of this model in case of simplified rout prudential value or lower bound adjustment is prescribed to be taken at the level of 0.1% of the total basic value level.

Proper comparison with results which gives developed approach should be done for adequate conditions, including level of certainty which is also fixed at CRR/EBA regulations at the level of 90%. Growth tendency is not considered in the referenced regulatory documents, hence for the sake of proper comparison we should assume of stagnating market example where growth factor μ = 0. Based on these initial assumptions developed analytical model gives time factor $\Delta_{t,p,prud}$ = 26.20 $/sq. m. or 2.4% of basic market value. As could be seen results which could be received from the model developed are sensitively above of those prescribed by the European regulations for such cases.

One of the possible reasons for such difference is much higher level of volatility which is inherent to the Ukrainian residential property market which is at least 2.5 times higher in comparison to the developed economies [43]. Such difference in value evolution stochasticity may have direct influence on the level of the lower boarder of the process evolution through parameter σ which reflect this factor in the model developed. For growing markets with growth factor μ>0 correction factor $\Delta_{t,p,prud}$ is decreasing being still above of the level of prudent value adjustment fixed by the CRR/EBA Regulation (Table 1).

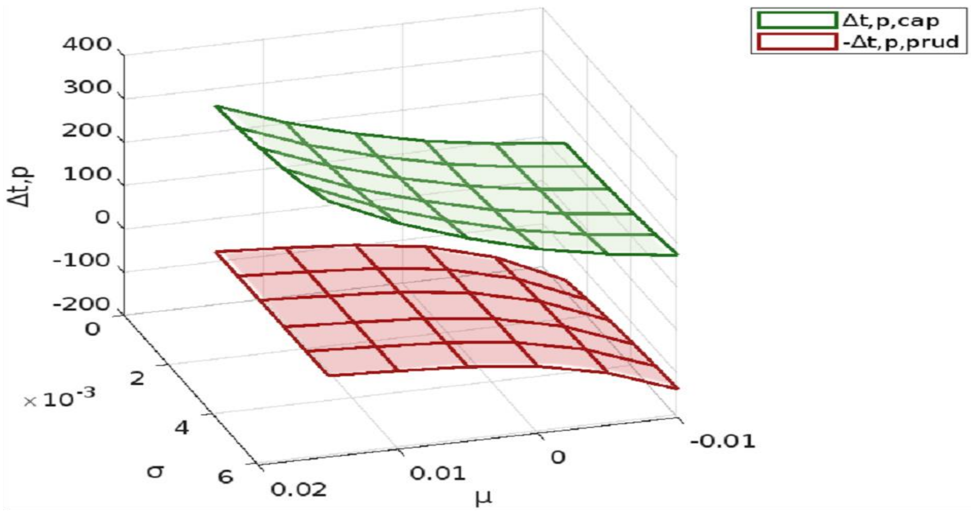

Figure 6. Three-dimensional dependence of the $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ parameters on the variance σ and growth coefficient μ for the upper and lower bounds.

Source(s): Author's own creation

Summarizing stated above, general model of stochastically based analytical approach developed for time effect consideration in assets valuation processes to address both value upper or “cap” and lower or “prudent” bound could be demonstrated by Figure 3. The main task is to estimate, based on methodology developed, the level of correction parameters $\Delta_{t,p,cap}$ and $\Delta_{t,p,prud}$ for individual asset subject of analysis using volatility level σ from market historical data registered, with expected market growth μ and reliability level p required.

Following the simple equations (9) for the long-term cap and prudent values estimation, this correction parameters should be added for upper bound and deducted for lower bound from the asset market value estimated by the conventional valuation methods to satisfy, in particular, the statement of the IVSC/TEGoVA/RICS joint research group, which underlined that "any prudent value assessment should be accompanied by the market value wherever possible, as this is a necessary benchmark requirement to accord with Basel III definition" [24].

7. Prospects for further research development

Overall methodology of upper or "cap" and lower or "prudent" value estimation under approach developed can be split in the following sequential steps:

- to estimate by any appropriate conventional method asset market value V_o at time point t=0;
- based on historical data estimate asset market value V_t volatility level σ;
- based on historical data and forecast estimate asset value expected growth factor μ;
- to accept certainty/reliability level p needed;
- to calculate correction parameters $\Delta_{p,t,cap}$, $\Delta_{p,t,prud}$ whichever are required;
- to add correction parameter $\Delta_{p,t,cap}$ to the market value V_o to get a long-term "cap" value or to deduct correction parameter $\Delta_{p,t,prud}$ from the market value V_o to arrive to asset "prudent" value estimation.

As a result being based on theoretical consideration of individual assets volatility which is coming from Samuelson's Rational Theory of Warrant Pricing developed approach with given assumptions and limitations underlined above as for accepted level of certainty expressed by probability p, process evolution volatility σ and general tendency μ could be used for estimation both upper or "cap" and lower or "prudent" asset value at any moment of time horizon with given level of its market value (Fig.2). Being quite different from existing "through-the-cycle" and "under-the-cycle" approaches, the novel proposed methodology could be referred as "follow-the-cycle" model which provides the estimate of long-term extreme values of the asset.

8. Conclusions

To summarize the main points considered, it could be stated that the problem of reliable and trustable valuation methods should take into consideration the assets value stochasticity evolving over time under the influence of different factors spectrum. This problem is directly linked with stability and transparency of financial system as underlined, in particular, by the Basel III Accord and European Banking Authority recommendations, which determined necessity for valuation practice to exclude application of market value or fair value concept solely. Such thesis was strongly advocated in recent summarizing research report funded jointly by the Investment Property Forum and the Property Research Trust [24]. Moreover, need for "… valuation methodology for providing a prudential value in a real estate context" was underlined in the IVSC special letter devoted to this issue [44].

In intention to compensate this gap a novel stochastic approach to volatility consideration in assets valuation processes is developed, which is based on Samuelson's Rational Theory of Warrant Pricing. Analytical solutions for assessment of proposed correction factors for upper or "cap" and lower or "prudent" asset value bound have been received with reference to the market growth expected and reliability required. Based on that general structured methodology assessment of upper and lower asset extreme values over market time horizon evolution is proposed.

Main advantages of the model developed are coming from its particulars which include:

- stochastic analytical background based on well-established and proven to be efficient Samuelson’s Rational Theory of Warrant pricing avoiding schematic presentation of property value evolution over time used in existing most widely used approaches;
- in contrast to existing approaches being a general model it presents a common methodology to estimate both upper or “cap” and lower or “prudent” bound level of property value evolving stochastically over time;
- analytically based methodology to estimate property upper or “cap” value bound was not identified in a valuation technical literature to the best authors knowledge;
- model includes limited number of essential parameters which reflect stochastic behavior of property evolution over time, i.e. evolution trend through parameter μ, level of volatility through coefficient of dispersion σ and level of probability p;
- estimation of correction parameters Δt,p, cap, Δt,p, prud for upper or “cap” and lower or “prudent” bound value levels respectfully is based on direct analysis of value stochasticity trend and volatility level through coefficients μ and σ, accordingly;
- with the absence of initial market data base to calculate these 2 main parameters of the model they can be accepted based on property historical data in relation to property value evolution with its proper forecast when 3rd parameter of the model which is probability level p should be taken based on results reliability needed.
- main model concept is oriented to estimate correction parameters for upper or “cap” value bound and lower or “prudent” value bound to adjust the property market value Vo and arrive to its extreme magnitudes for the given period of time. It secures utilization in the model basic notion of market value adding analytically grounded approach to estimate its upper and lower boundaries based on actual data of value change stochasticity analysis.

Implementation of developed approach was successfully verified using Ukrainian residential apartments market data set available. Robust comparison with results which could be received under CRR/EBA regulations provided demonstrated reasonable adequacy of the developed model which should be additionally tested for different markets and economic-financial sectors.

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